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MODERN SPECTRAL METHODS WITH APPLICATIONS TO ALBERTA CLIMATIC DATA

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "Modern Spectral Methods with Applications to Alberta Climatic Data", submitted by Philippe Andre Lachapelle in partial fulfilment of the requirements for the degree of Master of Science in Meteorology.

Dedicated to

Arlene

for her love, patience and understanding

in our first year.

ABSTRACT

A comprehensive computer program is developed for two methods of spectral analysis.

The Fast Fourier Transform is used as the basis for the modified periodogram method. This method is found to be highly efficient. It is used as the primary technique for evaluating Alberta climatic data.

The Maximum Entropy Method of spectral analysis is investigated and found to be lacking in ease of use and accuracy. This method is intended to extract very long periods present in a data set. The unavailability of unbiased data casts doubt on the value of this method for climatological investigations.

An evaluation is made of the climatic data available on microfiche from the Atmospheric Environment Service, Fisheries and Environment Canada.

Eight Alberta stations are chosen for the study. A detailed history of the changes of the meteorological site is compiled for each station.

A five-year periodicity is found to be statistically significant in the fall temperatures of all eight stations. Also, three- and four-year periodicities are found to be significant in the winter and spring temperatures, respectively, of a majority of the eight stations.

A thorough investigation is made of the five-year period but no evidence is found to indicate that it could result from causes other than true climatic fluctuations.

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Mr. B. Siemens - Fort Vermilion Experimental Farm

Mr. E.H. Hobbs - Lethbridge Research Station

Mr. H. Hall - Beaverlodge Research Station.

This study was conducted while on Educational Leave from the Atmospheric Environment Service, Fisheries and Environment Canada.

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TABLE OF ABBREVIATIONS

ACV	Autocovariance Method
AES	Atmospheric Environment Service
ASL	Above Sea Level
BEA	Beaverlodge CDA
CDA	Canadian Department of Agriculture Research Station
DFT	Discrete Fourier Transform
FFT	Fast Fourier Transform
FPE	Final Prediction Error
FVR	Fort Vermilion CDA
GMT	Greenwich Mean Time
HQB	Headquarters Building
LAC	Lacombe CDA
LTH	Lethbridge CDA
MAN	Manyberries CDA
MEM	Maximum Entropy Method
RAN	Ranfurly
WRO	Western Regional Office of AES
YXD	Edmonton City
YXH	Medicine Hat

CHAPTER 1

INTRODUCTION

1.1 Periodicity or Fantasy?

The search for periodic fluctuations in the Earth's climate has been pursued for many years. The motivation for such research has centred around the hypothesis that the climate of our planet may repeat itself at more or less regular intervals. Should such a pattern exist, its discovery would contribute greatly to long-range predictions. However, we may be deluding ourselves at times by accepting too quickly results which appear at first sight to be "statistically significant".

A favoured approach has been to try to relate the fluctuations of the climate to the fairly regular periodicity of the Zurich sunspot number. A brief summary of many such studies was compiled by Lawrence (1965). Most of the studies reviewed in this paper involved graphical methods. Some studies used a visual correlation technique which is, at best, a most unscientific approach, while other studies considered only very brief periods and gave no indication of the extent of correlation outside such periods.

Better methods of analysis have been developed

following the advent of modern computers. However, this has not resolved the argument over the sunspot cycle and its relation to weather and climate. A good example of this argument is shown in articles written by Shapiro (1975) and Bain (1976). Both authors analyzed a 315-year record of central England temperatures compiled by Manley (1974). Shapiro found no significant evidence of sunspot-related periods while Bain, using a different analysis technique called the Maximum Entropy Method, found evidence of the 22-23 year double sunspot cycle. The method used by Bain is also used in this study. However, its results are very difficult to assess, as will be seen later, and should not be taken at face value.

Many studies in which time-series analysis was used with considerable care have been carried out over the last twenty years. A brief list of such studies would include Landsberg et al. (1959), Julian (1971), Joseph (1973a, 1973b), Trenberth (1975), Bunting et al. (1975), Bradley (1976), Jones and Kearns (1976), Madden (1977), Dyer and Tyson (1977).

A recent article by Georgiades (1977) has provided evidence of a four- to five-year cycle in the annual temperature of selected stations in the central Canadian Prairies. This result will be discussed at a later stage.

1.2 The Study

The present study attempts to combine sound statistical

methods, recently developed methods of spectral analysis, the renewed interest in climatic change, and an analysis of the climatic conditions over one of the more important agricultural regions of Canada.

Based on the numerous studies mentioned in the previous section, there seems to be little hope of finding evidence of true periodic phenomena in temperature and precipitation data. However, a study of the distribution of variance in the frequency spectrum of these data may show groups of frequencies which contribute significantly to the total variance. Conversely, a uniform distribution of variance across the frequency spectrum would suggest that there is little hope of using climatological information for producing long-range forecasts.

Care must be taken should an apparently significant result be found. Such a result may be artificially induced in the data by factors such as the data processing method, changes of location of the observing site, aliasing of frequencies beyond the Nyquist frequency, or leakage of power from the low frequencies.

To date, most of the published research has made use of the well-known autocovariance method of spectral analysis. However, important new spectral methods have been revived or developed within the last ten years. These are the modified periodogram analysis using the Fast Fourier Transform and the Maximum Entropy Method of spectral analysis. Each of these two methods has distinct advantages over the

autocovariance method.

The Fast Fourier Transform approach produces a considerable saving of computational time while providing as much detail in the frequency distribution as is available when using the ACV method. The Maximum Entropy Method offers the potential of resolving very low frequencies, given data which contain only one or slightly less than one cycle of the pertinent low frequency. Both methods will be discussed in greater detail in later chapters.

Rikiishi (1976) made a detailed comparison of the modified periodogram and autocovariance methods and found that the two methods are theoretically equivalent. However, the modified periodogram permits direct calculation of the phase angle, while the autocovariance method requires a separate Fourier analysis in order to derive the phase. Also, his calculations of the computational time required were based on a comparison between a heavily smoothed periodogram and an autocovariance function obtained using a fairly small maximum lag. Although smoothing is applied to the periodogram in this study, the unsmoothed results are of equal interest since they can be analyzed without being concerned about the effects of the response function of the filter. A significant increase of computational time is required to obtain as much detail from the autocovariance method as is available from the unsmoothed periodogram. Thus, the modified periodogram was preferred.

1.3 Weather Parameters

As in any study of this type, the availability and homogeneity of the basic data provide many concerns.

The basic data collected for the study are the published monthly mean temperatures and the total monthly precipitation amounts (converting snow amounts to water equivalent) at eight Alberta stations. The selection procedure is described in Chapter 2 along with an assessment of accuracy and completeness.

One of the major problems in climatic data analysis is inhomogeneity resulting from changes of the instrument site. Some of these changes, such as installing a different type of thermometer or rain gauge, or a change of the observer, are generally impossible to determine prior to the 1950's since comprehensive records of such changes were ill-kept before that time.

Changes of location or of the surrounding obstacles are believed to be of greater significance and can usually be ascertained after a certain amount of probing. The changes of location of the eight stations used in this study have been determined and are discussed in Chapter 3.

Different groupings of the data are considered in this study. They consist of:

1- Using all values at one-month intervals for each parameter to determine the frequency distribution for frequencies greater than one cycle per two months. This analysis requires an appropriate filter in order to suppress the annual cycle.

2- Using the data for each month of the calendar year in order to determine variations in the individual months, for example, to determine the frequency spectrum of January mean temperatures or January precipitation amounts.

3- Using seasonal means in order to determine variations for each season. The frequent occurrence of convective precipitation in the summer in Alberta will likely make this season difficult to assess.

A comparison of results for each station should also be considered. Such an analysis may shed some light on the areal extent of significant results, i.e., it may indicate whether apparent cycles are strictly a local phenomenon or whether they can be identified over certain areas of the province if not over all stations involved. However, erroneous conclusions resulting from data processing procedures or limited sample sizes common to all stations would not necessarily be revealed by such analyses.

CHAPTER 2

DATA

2.1 Selection

Within the province of Alberta, there are a number of locations which have weather records covering periods of the order of 70 to 100 years.

The first step was to determine which weather or climatological stations had kept records that combined the following criteria:

1- Length of records exceeding 50 years.

2- Complete temperature and precipitation records.

3- Insignificant or no change of location of the observing site during the length of the record.

The most convenient source that provided this information is the Climatological Station Data Catalogue (1976). A total of 27 stations was chosen as a possible set of benchmark stations for the project; very few of these 27 stations met all three criteria but, of the over 600 stations listed in the catalogue, these 27 stations came closest to meeting the criteria. As will be explained in Chapter 3, this catalogue is not a fully reliable source of information but it does provide an excellent guide for making an initial choice.

The next step was to examine the records available. This was readily done using the microfiche station records available at the Western Regional Office (WRO) of the Atmospheric Environment Service (AES) located in Edmonton.

The WRO has available, on microfiche, the complete monthly record of every station in Alberta up to December 1972. Each microfiche plate provides such items as monthly mean maximum, mean minimum and mean temperature, extreme temperatures for the month, total monthly precipitation, rainfall and snowfall, and many other monthly values.

Of interest were the monthly mean temperature and the total monthly precipitation. A scrutiny of the microfiche records provided immediate and detailed information as to the completeness of the records. This process reduced the total number of suitable stations to 19.

Copies of the pertinent information were obtained for these 19 stations. Values for the period after 1972 were obtained from the Monthly Record (1973-1975), published by the AES.

Of the 19 stations, 8 were chosen as the main stations to be used in the project. They are:

1- BEA - Beaverlodge CDA (Canadian Department of Agriculture) - 60 years (of records).

2- YXD - Edmonton City - 93 years.

3- FVR - Fort Vermilion CDA - 68 years.

4- LAC - Lacombe CDA - 68 years.

5- LTH - Lethbridge CDA - 68 years.

6- MAN - Manyberries CDA - 47 years.

7- YXH - Medicine Hat - 91 years.

8- RAN - Ranfurly - 71 years.

Although MAN falls short of the 50-year criterion, it was chosen because of the completeness of its record; the value of such a record will become evident later on.

2.2 Verification

In order to determine the accuracy of the data kept on microfiche, a brief comparison of these data was made with the published values in the Monthly Record (1880-1972). This comparison revealed a number of discrepancies. As no one at the WRO could provide definite information concerning the accuracy of the microfiche data, a comprehensive comparison of data was undertaken to determine the extent of these discrepancies. Where available, daily values were used to recalculate the monthly values when these values did not agree with the microfiche record.

After a number of discrepancies were found, a letter was sent to the Information Section of the Headquarters of the AES in Toronto, Ontario, requesting verification of the microfiche data in cases where it did not agree with the published monthly values. The comparisons that were undertaken and the results are shown in Table 2.1. It should be noted that tolerances of .2F (.11C) for temperature and .02 inches (.5mm) for precipitation were allowed since earlier data would have been recalculated by computer

resulting in slight differences due to round-off of daily and monthly values.

Table 2.1 shows that of 350 discrepancies found by comparing 8565 values, 88 were verified and all findings supported the values given in the microfiche record. Of the 350 discrepancies, 207 were for differences greater than one degree Fahrenheit (.56C) or one tenth of an inch (2.5mm) of precipitation; 61 of the 88 values resolved were also in this category, with some differences being as large as one order of magnitude or more. Also, 103 of the 350 discrepancies occurred in the very early data (prior to 1891). The breakdown of these 103 values is contained in Table 2.1, where the values in parentheses represent the period prior to 1891 and are included in the number given for the total period of comparison.

As a result of these findings, Mr. Derek Aston was contacted in Toronto. He explained the procedure used to compile the microfiche records which are available for most stations across Canada.

The process began in the early 1960's and required several years to complete. Trained Meteorological Technicians verified the original weather records for each station up to the time when comprehensive verification of data became routine. Errors and conflicts found in the original records were resolved using all available information and the technicians' knowledge; errors and conflicts which could not be resolved satisfactorily

Table 2.1. Summary of data verification. T - Temperature, P - Precipitation. ( ) - Period before 1891. (see text for interpretation.)

Data	Set	Period	Total Verified	Conflicts Found	Resolved using: Daily Values	Resolved using: Toronto Records	Results favoring: Micro-fiche	Results favoring: Monthly Record
BEA	T	Sep/1915-Dec/1950	424	2	1	0	1	0
	P	Apr/1915-Dec/1950	429	20	0	0	0	0
YXD	T	Sep/1881-Dec/1950	832	48 (30)	2	7 (3)	9 (3)	0
	P	Apr/1882-Dec/1950	825	32 (5)	6	11 (5)	17 (5)	0
FVR	T	Jul/1908-Dec/1950	510	18	7	1	8	0
	P	Jul/1908-Dec/1950	510	21	2	4	6	0
LAC	T	Dec/1907-Dec/1950	517	11	3	0	3	0
	P	Mar/1908-Dec/1950	514	21	4	1	5	0
LTH	T	Mar/1908-Mar/1939	372	9	2	1	3	0
	P	Feb/1908-Mar/1939	373	18	0	3	3	0
MAN	T	Sep/1928-Dec/1950	268	4	3	0	3	0
	P	May/1928-Dec/1950	272	6	1	0	1	0
YXH	T	Sep/1883-Dec/1950	808	91 (65)	7	7 (4)	14 (4)	0
	P	Oct/1883-Dec/1950	807	21 (3)	4	6 (3)	10 (3)	0
RAN	T	Jan/1905-Dec/1950	552	8	5	0	5	0
	P	Jan/1905-Dec/1950	552	20	0	0	0	0
Sub	T		4283	191 (95)	30	16 (7)	46 (7)	0
Total	P		4282	159 (8)	17	25 (8)	42 (8)	0
Total			8565	350 (103)	47	41 (15)	88 (15)	0

resulted in a missing value on the microfiche. Major differences between the new computed values and the original published information were again verified. The objective of this program was to produce data which had been processed uniformly for the entire length of the record and the final product was made available on microfiche.

Although there exists a slight possibility that errors may be present in the microfiche record due to key-punch errors, this author feels that the findings of the comparison carried out reduce the frequency of such errors to insignificant proportions. The time and effort involved in compiling the microfiche record have resulted in one of the most accurate amd homogeneous sets of weather records available.

2.3 Summary

Having accepted the microfiche records, the next step was to determine values for the few missing entries.

In carrying out the comparison, some of the values entered as missing on the microfiche were found in the Monthly Record. Some of these missing values were also investigated by Mr. Aston. However, no changes were made to the microfiche record, indicating that the available data for that month did not meet specified requirements.

First-guess estimates of the missing values were made using the values published in the Monthly Record or, if also missing in this source, by estimating a value based on other

Table 2.2. Summary of data sets. (T - Temperature, P - Precipitation.)

Data Set		Period	Total Number	Number Missing	Number Found*	Number Est.
BEA	T	Sep/1915-Dec/1975	724	0	0	0
	P	Apr/1915-Dec/1975	729	2	1	1
YXD	T	Sep/1881-Dec/1975	1132	4	0	4
	P	Apr/1882-Dec/1975	1125	1	0	1
FVR	T	Jul/1908-Nov/1975	809	0	0	0
	P	Jul/1908-Nov/1975	809	5	1	4
LAC	T	Dec/1907-Dec/1975	817	1	1	0
	P	Mar/1908-Dec/1975	814	6	3	3
LTH	T	Mar/1908-Dec/1975	814	0	0	0
	P	Feb/1908-Dec/1975	815	0	0	0
MAN	T	Sep/1928-Dec/1975	568	0	0	0
	P	May/1928-Dec/1975	572	2	0	2
YXH	T	Jul/1884-Dec/1975	1098	4	1	3
	P	Oct/1883-Dec/1975	1107	15	11	4
RAN	T	Jan/1905-Dec/1975	852	0	0	0
	P	Jan/1905-Dec/1975	852	5	4	1
Sub	T		6814	9	2	7
Total	P		6823	36	20	16
Total			13637	45	22	23

* Data 'Found' refers to data published in the Monthly Record but rejected when the microfiche data were compiled.

stations in the same area. Table 2.2 shows the total periods of data involved and the numbers of missing data values.

Only 45 values were missing out of a total of 13,637 from the eight stations chosen, and 34 of these occurred in 1916 or earlier. Thus, for the period 1917 to 1975, only 11 values were missing from a total of 11,050 values, and seven

of these were found in the Monthly Record.

Further discussion of the missing values will be made in Chapter 7. It suffices to say at this time that the data prepared for this study are believed to be as sound as possible.

CHAPTER 3

STATION HISTORIES

3.1 Sources of Information

Any statistical analysis of meteorological information must take into account changes of location of the observing sites. The listings in the Climatological Station Data Catalogue provide a history of each station in terms of latitude, longitude and elevation. However, comprehensive on-site inspection by AES of observing stations did not begin until the early 1950's; thus earlier knowledge of station locations is often very meager and subject to much doubt. An appeal for improvement was recently made by Catchpole and Ponce (1976).

Since the Catalogue showed no change in the location of Medicine Hat since 1883, doubt was immediately cast on the accuracy of its listings; it did not seem likely that the present Medicine Hat Airport would be located at the original observing site.

In order to eliminate as much of this doubt as possible, a thorough search of all information available was made. The best local sources of information are the inspection report files at the WRO. These files provided good histories of Medicine Hat and Ranfurly. A history for

Edmonton was readily available in a separate source. As for the CDA stations, little information could be gained from the files so that letters were sent to these places requesting historical information if available. All replies were very complete and most included maps showing the exact changes of the meteorological sites.

The history of Medicine Hat, as found in the WRO files, was not totally clear. There were references to commercial establishments, some of which are no longer in existence. A visit to the Alberta Government Telephones archives was necessary to determine the exact locations of the meteorological sites.

The remainder of this chapter outlines the history of each station. The possible consequences of observing site changes will be discussed along with the numerical results in Chapter 7.

Appendix A contains a topographical map for each station showing the various positions of the observing sites.

3.2 Beaverlodge CDA

A letter received from Mr. H. Hall, Agricultural Meteorological Technician at the experimental farm provided detailed information on the Beaverlodge site.

The first site, at an elevation of 750m (2460 feet), was to the North of the present site at an elevation of 739m (2425 feet); the distance between the two sites is 373.7m

(1226 feet). The terrain in this area is slightly rolling.

Mr. Hall also advised that prior to the official move of the observing site in 1958, a three-year comparative study was made. Although the original records could not be found, this study, referred to by Carder (1962), will be discussed in Chapter 7.

The official position of the present site is 55°12' North 119°25' West at an established elevation of 731.5m (2400 feet) above sea level (ASL). (Established position and elevation may differ from actual values because of the scale of the topographical map used; WRO uses maps with a scale of 1/250,000.)

3.3 Edmonton City

The history of Edmonton was readily available in the Annual Meteorological Summary for Edmonton (1974). As data to be used start in September 1881, the history prior to this date is presented for completeness only.

The following text is taken from the Annual Summary:

> History
>
> On December 29, 1879, Mr. G.S. Wood, a telegrapher, completed installation of a meteorological station at Fort Edmonton. Instruments included two standard thermometers, one maximum and one minimum thermometer, a rain gauge, one (common) thermometer screen and shed, and a Forsters Small anemometer with double tail vane. On May 19, 1880, a Greens Small Barometer, No.2389, was first used.
>
> Daily observations commenced July 11, 1880, with minor interruptions when Mr. Wood was absent repairing the telegraph line. Evidently, the anemometer was set up outside the Fort, because several evening observations of the wind are

missing with the notation: "Fort gate locked prematurely; could not get out to take wind gauge".

In 1882, the meteorological station was turned over to the Hudson Bay Company. Mr. C.F. Young and his brother took the reports at a site just north of Jasper Avenue on what is now 115th Street. This is just 3.2km (2 miles) south of the present weather station and is about the same altitude.

In April 1912, Mr. S.M. Holmden became responsible for the observations and the site was changed to 63rd Street in the Highlands. Altitude of this station was only 7.6m (25 feet) lower than the present Airport site.

Mrs. E. Owen took over the observing duties in 1915 and continued in this post till the climatological station was closed in 1942. (Meteorological reports for Edmonton use the data collected at the Airport since 1937.)

Present Location

The approximate location of the Airport is 53°35' North, 113°30' West. The official elevation of the Airport is 670.5m (2200 feet). The Airport is surrounded by the City of Edmonton and is approximately 4.8km (3 miles) northwest of the city centre. The meteorological office was first moved to the Airport as a permanent arrangement approximately the first of September, 1937. It was then located in what is now the Western Airmotive Hangar. Approximately the first of November, 1942, it was moved to the Airport Administration Building.

A new terminal building has been completed and the Weather Office was relocated early in 1976; however the data base ends with December 1975 and is not affected by this latest move.

3.4 Fort Vermilion CDA

A letter received from Mr. B. Siemens, the Superintendent of the experimental farm, relayed the following information:

> From 1908 until the winter of 1935-36, the site was located on a river flat on the south bank of the Peace River.... at an estimated elevation of 259m (850 feet). I believe that this meteorological site was in an open area well away from bush.
>
> Since 1936, the meteorological site has been on the Experimental Farm,.... at an elevation of 277m (910 feet). The site is in an open field.

The move was approximately 7.65km (4.75 miles) eastwards. The present location of the station is 58°23' North, 116°02' West, and the established elevation is 279m (915 feet) ASL.

3.5 Lacombe CDA

A letter received from Mr. J.R. Gillespie, the Meteorological Observer at the farm, advised that the observing site had been moved only once in its history.

The site was moved 1.15km (0.72 mile) southwestwards to a more open area. This move was made on July 1, 1972.

The terrain in this area is very level. Thus the move resulted in a negligible change of elevation (approximately one meter lower than the previous location).

The official position of this site is 52°28' North, 113°45' West, and its established elevation is 847m (2780 feet) ASL.

3.6 Lethbridge CDA

A letter was received from Mr. E.H. Hobbs, a research scientist at the CDA, outlining the history of the meteorological site.

Since observations began in 1908, the site has been moved only once - in June of 1966. The move was approximately 602m (1975 feet) southwestwards, with negligible change in elevation (a decrease of 3.4m over very flat terrain).

The present location is 49°42' North, 112°47' West, and the established elevation is 899m (2950 feet) ASL.

3.7 Manyberries CDA

Manyberries CDA is a substation of Lethbridge CDA. Mr. E.H. Hobbs, in his letter about Lethbridge CDA, outlined the history of the meteorological station at Manyberries CDA.

Since observations began in 1928, the site has been moved twice. Both moves were done because the shelterbelt had grown too high over the years.

The original location was a few hundred meters west of the Headquarters Building (HQB). In 1944, the site was moved to a location 129.5m (425 feet) east of the HQB; and in 1949, the site was moved to a location 259m (850 feet) northnortheast of the HQB resulting in an elevation increase of 7.6m (25 feet).

The present location is 49°07' North, 110°28' West, and the established elevation is 934m (3065 feet) ASL.

3.8 Medicine Hat

A newspaper article written in 1933 by an unnamed

author was found in the inspection files of the WRO. A concise history, taken from this article and updated to 1956, was also found in the files. The history is as follows:

> The first month for which observations are complete is that of September 1883. The observer was Mr. J.L. Ewart. The station was situated on Second Street on the site now occupied by Taylor Bros. (in 1933). The elevation was 665.3m (2183 feet) above mean sea level.
>
> In 1891, the station was taken over by Mr. J.K. Drinnen, and moved across the street to a point where the Cawker Drug Store now stands.
>
> Another change was made in 1901, when Mr. Walter Crosskill took over as observer. The station was located where the Moore furniture store is now situated (about 90m south of City Hall).
>
> In January 1911, Mr. H.H. Hassard was the observer. The station was moved to the corner of Fourth Street and Ash Avenue, where observations were taken until Mr. Hassard's death in 1929. Mr. W.H. Phelps became acting observer until August 1930.
>
> In August 1930, Mr. C. Pickering was appointed full time weather observer to set up and operate the weather station in connection with the opening of the Prairie Air Mail Service. The station was set up temporarily at the High School site where the elevation was 706.5m (2318 feet), then moved to the airport at an elevation of 720.8m (2365 feet) in the Spring of 1931. When the Prairie Air Mail was abandoned in March 1932, the weather station was moved back to the city to a location on Crescent Heights at an elevation of 703.1m (2307 feet).
>
> In 1933, the station was moved to a location immediately west of Division Avenue on Third Street at an elevation of 704m (2310 feet).
>
> The station was moved back to the Airport Radio Range station in 1938 at a location on the north side of the Airport which is located about 3.2km (2 miles) southwest of the city centre.

There are no records of any changes after 1938. In brief, the observing site was located in the South Saskatchewan River valley from 1883 to 1930, and it has

remained out of the valley since 1931.

The present location is 50°01' North, 110°43' West, at an established elevation of 720.8m (2365 feet) ASL.

3.9 Ranfurly

A brief history found in the inspection files at the WRO shows that the climatological program has been maintained by the same family since the first of January, 1905. Presently, the grandsons of the original observer are performing the duties.

The instrument site was moved about 0.8km (.5 mile) westwards in the Fall of 1931. Previous to this, the location was closer to brush and more sheltered. From the Fall of 1931 to June 18, 1951, the thermometer screen was situated in a more sheltered location, some 12m (40 feet) to the east of its present site. The move in 1931 resulted in no appreciable change in elevation.

The position of the station is 53°27' North, 111°39' West, and the established elevation is 685.8m (2250 feet) ASL.

CHAPTER 4

FAST FOURIER TRANSFORM

4.1 Theory

The Fast Fourier Transform (FFT), first developed in the early 1900's, returned to prominence in the mid 1960's as a method capable of saving considerable computational time (Cochran et al., 1967; Bingham et al., 1967; Cooley et al., 1967).

The basic formula for the transform is

$$y_j = \sum_{k=0}^{q-1} x_k e^{-i2\pi kj/q} \qquad (4.1.1)$$

where x_k represents a time function, y_j represents the transform, and $j=0,1,\ldots,q-1$.

Equation (4.1.1) forms the basis of the FFT even though it is the definition of the Discrete Fourier Transform (DFT).

In calculations of the FFT, the data set is factored into multiple components of two before using (4.1.1). Using matrix notation, the procedure, as described by Robinson (1967), consists of taking two 1xq row vectors X and Y and a qxq matrix W to obtain

$$Y = XW \qquad (4.1.2)$$

where $X = [x_0, x_1, \ldots\ldots, x_{q-1}]$,

$$Y = [y_0, y_1, \ldots\ldots, y_{q-1}],$$

$$W = [e^{-i2\pi kj/q}] \qquad k, j=0, 1, 2, \ldots\ldots, q-1.$$

The matrix W is symmetric and has the property that

$$WW^{*T} = qI. \qquad (4.1.3)$$

The FFT algorithm factorizes the matrix W into n sparse matrices $S_1, S_2, \ldots\ldots, S_n$ and a permutation matrix P such that

$$W = S_1 S_2 \ldots\ldots S_n P \qquad (4.1.4)$$

where $n = \log_2 q.$

The sparse and permutation matrices can be illustrated for $q=2^3=8$. If we define w as

$$w = e^{-i2\pi/q},$$

then the matrix W can be written as

$$W = [w^{kj}] \qquad k, j=0, 1, 2, \ldots\ldots, q-1.$$

Thus we have

$$W = \begin{bmatrix} w^0 & w^0 & w^0 & w^0 & w^0 & w^0 & w^0 & w^0 \\ w^0 & w^1 & w^2 & w^3 & w^4 & w^5 & w^6 & w^7 \\ w^0 & w^2 & w^4 & w^6 & w^8 & w^{10} & w^{12} & w^{14} \\ w^0 & w^3 & w^6 & w^9 & w^{12} & w^{15} & w^{18} & w^{21} \\ w^0 & w^4 & w^8 & w^{12} & w^{16} & w^{20} & w^{24} & w^{28} \\ w^0 & w^5 & w^{10} & w^{15} & w^{20} & w^{25} & w^{30} & w^{35} \\ w^0 & w^6 & w^{12} & w^{18} & w^{24} & w^{30} & w^{36} & w^{42} \\ w^0 & w^7 & w^{14} & w^{21} & w^{28} & w^{35} & w^{42} & w^{49} \end{bmatrix};$$

but $w^{24}=w^{16}=w^8=1$, $w^{36}=w^{28}=w^{20}=w^{12}=w^4$, etc., and

$$W = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & w & w^2 & w^3 & w^4 & w^5 & w^6 & w^7 \\ 1 & w^2 & w^4 & w^6 & 1 & w^2 & w^4 & w^6 \\ 1 & w^3 & w^6 & w & w^4 & w^7 & w^2 & w^5 \\ 1 & w^4 & 1 & w^4 & 1 & w^4 & 1 & w^4 \\ 1 & w^5 & w^2 & w^7 & w^4 & w & w^6 & w^3 \\ 1 & w^6 & w^4 & w^2 & 1 & w^6 & w^4 & w^2 \\ 1 & w^7 & w^6 & w^5 & w^4 & w^3 & w^2 & w \end{bmatrix}.$$

The order of the powers of w in the sparse and permutation matrices is determined by the reverse binary values of 0,1,2,3,4,5,6,7 which are 0,4,2,6,1,5,3,7.

This yields

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ W^0 & 0 & 0 & 0 & W^4 & 0 & 0 & 0 \\ 0 & W^0 & 0 & 0 & 0 & W^4 & 0 & 0 \\ 0 & 0 & W^0 & 0 & 0 & 0 & W^4 & 0 \\ 0 & 0 & 0 & W^0 & 0 & 0 & 0 & W^4 \end{bmatrix},$$

$$S_2 = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ W^0 & 0 & W^4 & 0 & 0 & 0 & 0 & 0 \\ 0 & W^0 & 0 & W^4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & W^2 & 0 & W^6 & 0 \\ 0 & 0 & 0 & 0 & 0 & W^2 & 0 & W^6 \end{bmatrix},$$

and

$$S_3 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ W^0 & W^4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & W^2 & W^6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & W & W^5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & W^3 & W^7 \end{bmatrix}.$$

Thus the algorithm requires that the number of data points be a power of two. If the number of values in the data set is not a power of two, zeroes must be added until the total number q meets this requirement.

Once W has been broken down into its many component matrices (4.1.4), the computational procedure becomes much shorter; in fact the computing time required is $q(\log_2 q)/q^2$ of the time required when using the DFT on the full data

set.

All computations are done using complex functions such that, for real-valued x_K, the cosine amplitude is

$$Re(y_j) = \sum_{K=0}^{g-1} x_K \cos(2\pi K j/g) \qquad (4.1.5)$$

and the sine amplitude is

$$Im(y_j) = -\sum_{K=0}^{g-1} x_K \sin(2\pi K j/g) \quad . \qquad (4.1.6)$$

It is easily seen that for frequency zero and the Nyquist frequency, the imaginary part of the amplitude will be zero when using real data and an FFT technique based on a power of two. Also, the real part of the amplitude is symmetric about the Nyquist frequency, $f_N = \frac{1}{2\Delta t}$, such that

$$Re(y_{g/2-r}) = Re(y_{g/2+r}) \qquad \text{for } r=1,2,\ldots,\tfrac{g}{2}-1,$$

and the imaginary part is anti-symmetric about the Nyquist frequency such that

$$Im(y_{g/2-r}) = -Im(y_{g/2+r}) \qquad \text{for } r=1,2,\ldots,\tfrac{g}{2}-1.$$

The values for the cospectrum are calculated using

$$P_j = 2\{(Re(y_j))^2 + (Im(y_j))^2\} \quad . \qquad (4.1.7)$$

Since the data are standardized by removing the mean and dividing by the standard deviation prior to analysis, the square of the amplitude is recovered from (4.1.7) by using

$$a_j^2 = 2 P_j \frac{\sigma^2}{\sigma_x^2} \qquad (4.1.8)$$

where σ^2 is the variance of the q data values and $\sigma_x^2=1$ is the unit variance of the standardized values.

If, as is frequently the case, the number of values in the data set is not a power of two, a second multiplication factor must be used in order to compensate for the zeroes added; thus

$$A_j^2 = a_j^2 \frac{g}{N} \qquad (4.1.9)$$

where N is the number of values in the original data set.

Therefore, we have that

$$A_j^2 = \frac{2 P_j \sigma^2 g}{N} \qquad (4.1.10a)$$

or

$$A_j^2 = 2 P_j \sigma_o^2 \qquad (4.1.10b)$$

where σ_o^2 is the variance of the original N data values.

The corresponding phase and frequency are readily obtained by using

$$\phi_j = \tan^{-1}\left[\frac{Im(\gamma_j)}{Re(\gamma_j)}\right] \qquad (4.1.11)$$

and

$$f_j = \frac{j}{g\Delta t} \qquad (4.1.12)$$

where Δt is the time interval between data values and j=0,1,....,g/2. The phase angle ϕ_j refers to the phase lag of a cosine wave.

4.2 Filters

Two filters were programmed as options in calculating the FFT of the data. The first was a tapering filter

(Subroutine TAPER) which would taper the ends of the data sample in order to avoid pronounced discontinuities at the ends. The second was a Daniell filter (Subroutine DANI) which would smooth the output values for easier interpretation. The theory for both filters is described by Kanasewich (1975).

The tapering filter consists of a pair of cosine bells. The weights for one half of the filter are calculated using

$$V_m = \frac{1}{2}\left(1 + \cos \frac{\pi(m-L)}{L}\right) \qquad (4.2.1)$$

with m=0,1,....,L, and are applied using

$$Y_m = V_m x_m \qquad \text{and} \qquad Y_{N-m} = V_m x_{N-m}$$

where N is the total number of values and m=0,1,....,L.

The shape of the filter is given in Figure 4.1.

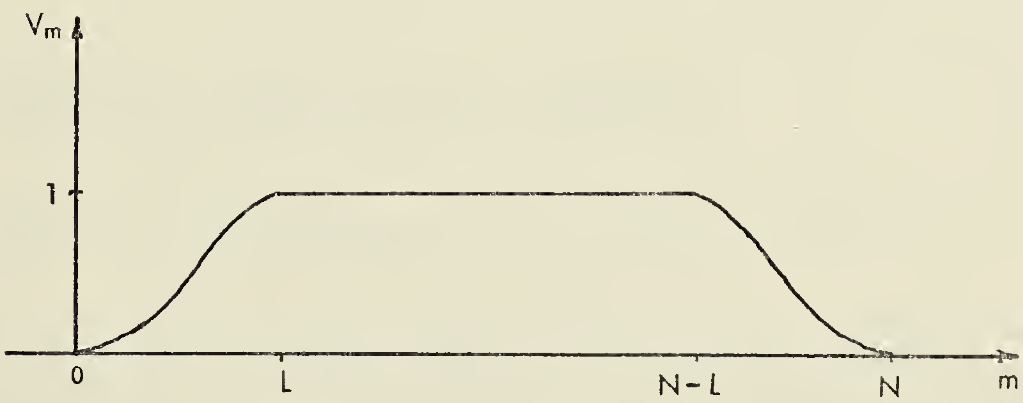

Figure 4.1. Taper filter.

The Daniell filter has a rectangular or box car shape response in the frequency domain as shown in Figure 4.2.

Kanasewich recommends using the Daniell filter on the periodogram calculated by the FFT since it reduces the

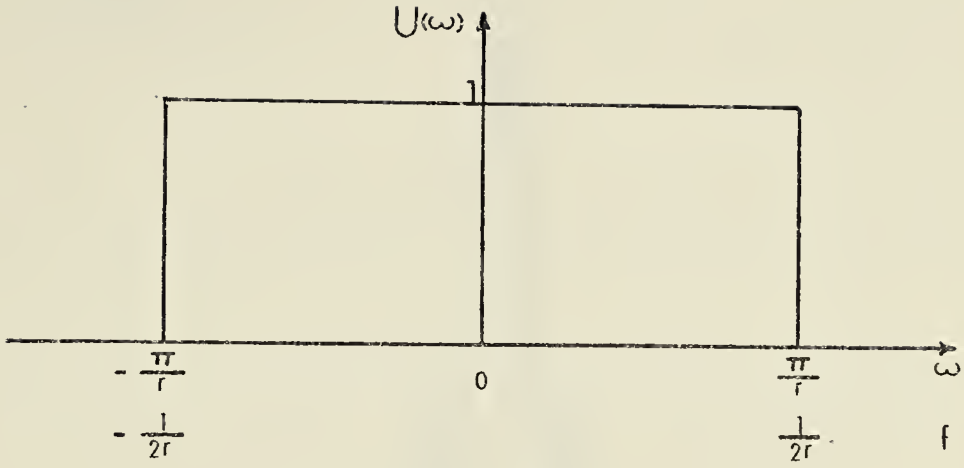

Figure 4.2. The Daniell or rectangular spectral window. (From Kanasewich, 1975.)

variance of the periodogram.

In order that q be a power of two, the data must be augmented by zeroes; this results in a smoothing of the spectral window as shown in Figure 4.3 for N/q=0.7 and 1.0, with r=7. The shape of the window is given by

$$H(\omega) = \frac{1}{2r+1} \sum_{j=-r}^{r} \frac{\sin^2\left(N\omega - \frac{2\pi Nj}{q}\right)}{2\pi r \sin^2\left(\omega - \frac{2\pi j}{q}\right)} \quad . \tag{4.2.2}$$

The spectral estimate now becomes

$$P(U) = \frac{q}{N}\,\frac{1}{2r+1} \sum_{j=-r}^{r} F(U-j)\,F^*(U-j) \tag{4.2.3}$$

where F is the FFT of the data, F^* is the complex conjugate of F and U=0,1,....,q/2. The factor 1/(2r+1) normalizes the window.

Special consideration must be given to the weighting factors when the filter is applied at or near the endpoints since the required number of values of the periodogram is not available to cover the full width of the filter. In

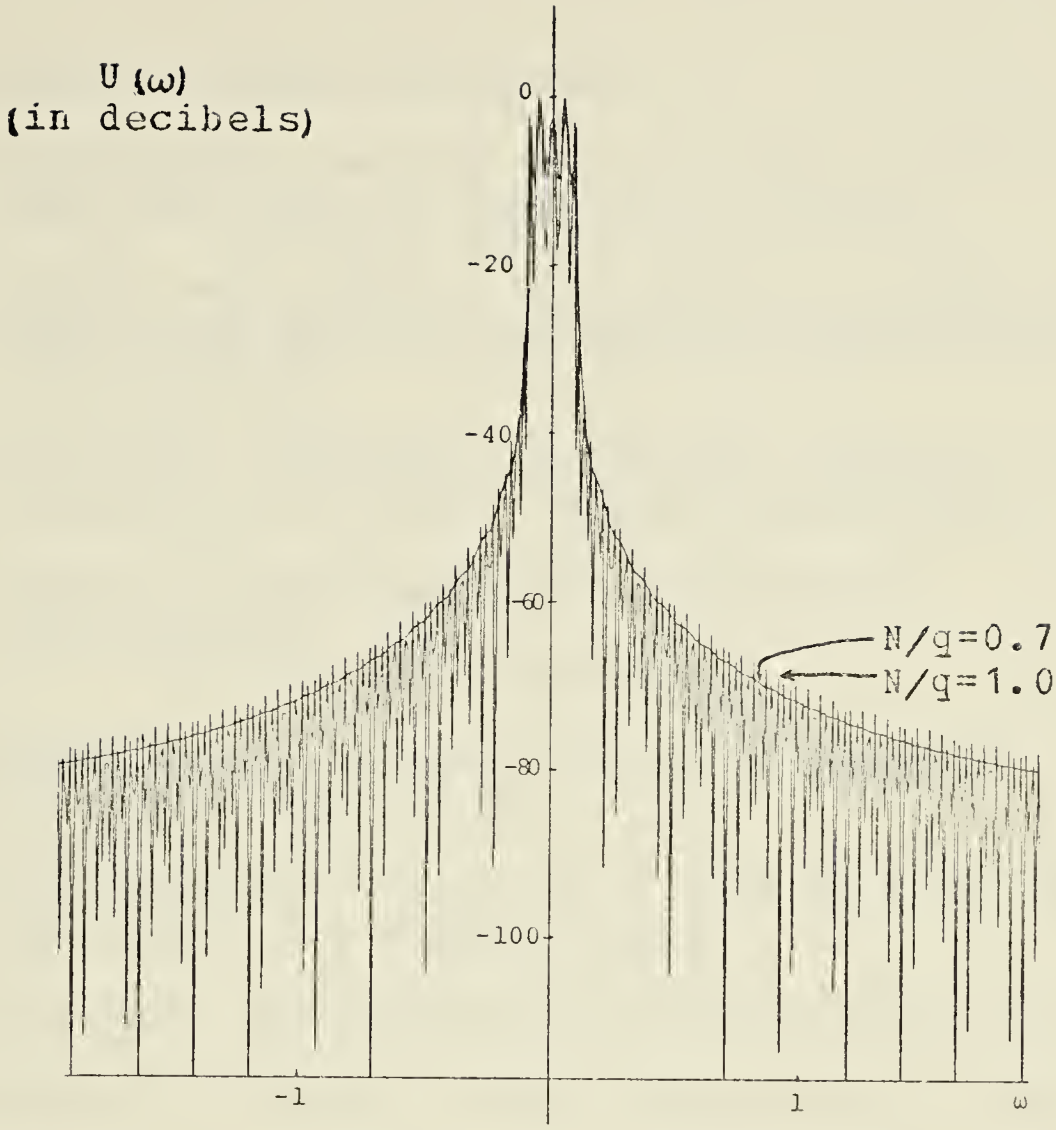

Figure 4.3. Daniell window for N/q=0.7 and 1.0, and r=7. (From Kanasewich, 1975.)

order to avoid this problem, the end points were ignored and the filter was applied only as needed to obtain the required number of values of the periodogram.

4.3 Programs and Data Manipulation

The prime subroutine used in the FFT analysis was FASTFT. The auxiliary subroutines required were TAPER, VARSD, NLOGN, POLAR, DANI and DANY; subroutine VARSD will be discussed in Chapter 6. All programs are listed in Appendix B.

Prior to the FFT analysis, the data were manipulated as follows:

1- The data mean was removed.

2- If desired, the data were tapered to zero at both ends using a cosine bell filter.

3- Zeroes were added at the beginning and/or the end of the data set until the desired size of 2^n was achieved.

4- The mean, variance and standard deviation were calculated for all 2^n values using VARSD.

5- The data were standardized using the relationship

$$y_i = (x_i - EM)/SD \qquad (4.3.1)$$

where EM is the mean, SD is the standard deviation, and $i=1,2,\ldots,2^n$.

6- The resultant values of y_i were assigned to the real part of a complex array X.

The actual calculations of the FFT were performed by the subroutine NLOGN. This subroutine appeared in Robinson (1967).

NLOGN calculates the FFT of the data and assigns the results to the complex array X. This subroutine is capable of calculating the reverse operation from transform to real data; this is accomplished by specifying the index of the exponential function to be +1.0. In calculating the transform, the index is specified as -1.0.

The complex array was broken down into its real and imaginary parts from which the power spectrum and phase were obtained using the subroutine POLAR. The subroutine POLAR was also obtained from Robinson (1967).

The square of the amplitude was calculated for each value in the power spectrum using (4.1.10).

If the results obtained at this step did not require

filtering, they were printed as shown in Table B.15 in Appendix B.

The real (COS) and imaginary (SIN) parts of the power spectrum and the complete power spectrum could be filtered using a Daniell filter (Subroutine DANI), as described in the previous section. The resultant smoothed values of COS and SIN were used primarily to calculate the phase values. The frequency was filtered in a slightly different manner by using a modified Daniell filter (Subroutine DANY); the power spectrum was used as a weighting factor in order to obtain a better estimate of the true peak frequencies. Both subroutines have a variable filter size as explained by the comment cards in the program listing.

All values obtained were printed (see Tables B.16 and B.17 in Appendix B).

CHAPTER 5

MAXIMUM ENTROPY METHOD

5.1 Theory

J.P. Burg (1967, 1968) proposed a new spectral analysis technique called the Maximum Entropy Method (MEM). The first publication discussing MEM appeared in 1967, at the same time as the publication promoting the FFT was released.

This technique was subsequently extended by other authors, in particular by Parzen, 1969 and Lacoss, 1971. T.J. Ulrych (1972) gave considerable impetus to MEM by obtaining amazingly good results with the technique. Some of these results were later challenged by Chen and Stegen (1974). However MEM does have desirable properties; the positive and negative aspects of this method will be examined in Chapters 7 and 8.

In very recent time, the use of MEM has found its way into meteorological research (Bain, 1976; Dyer, 1976; Mason, 1976).

As often happens, the emergence of something new is soon followed by variations on the main theme. Ulrych and Bishop (1975) provide two methods for calculating the spectral estimates using MEM, and Andersen (1974) presents a flow chart for one of these two methods.

Kanasewich (1975) discusses the method presented by Andersen (1974) and, as indicated in a personal conversation, recommends this approach.

MEM is a recursive method which bears greater resemblance to the autocovariance (ACV) method of spectral analysis than to the FFT method. The following description of MEM is taken from Chen and Stegen (1974), and Andersen (1974).

The entropy of a Gaussian band-limited time series is proportional to

$$\int_{-f_N}^{f_N} \log P(f)\, df \qquad (5.1.1)$$

where P(f) is the power spectrum and f_N is the Nyguist frequency. Based on the idea that the spectral estimate must be the most random (i.e. maximum entropy) of any power spectrum that is consistent with the measured data, Burg (1967, 1968) maximized the entropy shown in (5.1.1) through the use of Lagrange multipliers λ_n under the constraints of

$$\int_{-f_N}^{f_N} P(f)\, z^n\, df = \phi_n \qquad -N \leq n \leq N \qquad (5.1.2)$$

where $z = \exp(i2\pi f \Delta t)$, Δt is the sampling interval, and ϕ_n is the autocorrelation with time lag $n\Delta t$. Thus

$$\delta \int_{-f_N}^{f_N} \left\{ \log P(f) - \sum_{n=-N}^{N} \lambda_n \left[P(f) z^n - \frac{\phi_n}{2 f_N} \right] \right\} df = 0 \qquad (5.1.3)$$

which in turn yields

$$P(f) = \left(\sum_{n=-N}^{N} \lambda_n z^n \right)^{-1} \qquad (5.1.4)$$

where the Lagrange multipliers are to be determined by satisfying the constraints (5.1.2). Now, (5.1.2) is equivalent to

$$P(f) = \frac{1}{2f_N} \sum_{n=-N}^{N} \phi_n z^{-n} \quad . \qquad (5.1.5)$$

Since P(f) is real and nonnegative and $f_N = \frac{1}{2\Delta t}$, (5.1.4) can be rewritten as

$$P(f) = \frac{P_M \Delta t}{\left| 1 - \sum_{n=1}^{M} a_{Mn} e^{-i2\pi f n \Delta t} \right|^2} \qquad (5.1.6)$$

where M replaces N and is the lag value chosen to optimize the results.

MEM consists of calculating a set of filter coefficients a_{mn} and a constant P_m. From these values the power spectrum is estimated by (5.1.6) with f being limited to the Nyquist interval,

$$-\frac{1}{2\Delta t} \leq f \leq \frac{1}{2\Delta t} \quad .$$

P_m and the coefficients a_{mn} are determined by

$$\begin{bmatrix} \phi_0 & \phi_1 & \cdots & \phi_m \\ \phi_1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \phi_1 \\ \phi_m & \cdots & \phi_1 & \phi_0 \end{bmatrix} \begin{bmatrix} 1 \\ -a_{m1} \\ \vdots \\ -a_{mm} \end{bmatrix} = \begin{bmatrix} P_m \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (5.1.7)$$

so that P_m is the output power of the m+1 long prediction

error filter $(1,-a_{m1},\ldots,-a_{mm})$. Equation (5.1.7) is solved iteratively by stepwise increase of the matrix dimension from m-1 to m. For m=0, P_0 is estimated by

$$P_0 = \frac{1}{N}\sum_{t=1}^{N} x_t^2 \quad . \tag{5.1.8}$$

In general, after solution of (5.1.7) for m-1, the next step m involves the determination of a set of m+2 unknowns $(a_{m1},\ldots,a_{mm};\phi_m;P_m)$ from the m+1 equations.

According to Burg (1968) the following conditions should be used: the average output power of the m+1 prediction error filter

$$\Pi_m = \frac{1}{2(N-m)}\sum_{t=1}^{m}\left[\left(x_t-\sum_{k=1}^{m} a_{mk}x_{t+k}\right)^2+\left(x_{t+m}-\sum_{k=1}^{m} a_{mk}x_{t+m-k}\right)^2\right] \tag{5.1.9}$$

is minimized with respect to the single parameter a_{mm}. The dependence of the other filter coefficients on a_{mm} is determined by the m lower equations in (5.1.7) having the solutions

$$a_{mk} = a_{m-1\,k} - a_{mm}\cdot a_{m-1\,m-k} \tag{5.1.10}$$

for $k=1,2,\ldots,m-1$ and using $a_{m0}=-1$ and $a_{mk}=0$ for $k\geq m$.

Using (5.1.10), (5.1.9) can be written as

$$\Pi_m = \frac{1}{2(N-m)}\sum_{t=1}^{N-m}\left[\left(b_{mt}-a_{mm}b'_{mt}\right)^2+\left(b'_{mt}-a_{mm}\,b_{mt}\right)^2\right] \tag{5.1.11}$$

where

$$b_{mt} = \sum_{k=0}^{m} a_{m-1\,k}\,x_{t+k} - \sum_{k=0}^{m} a_{m-1\,m-k}\,x_{t+m-k} \quad , \tag{5.1.12a}$$

$$b'_{mt} = \sum_{k=0}^{m} a_{m-1\,k}\,x_{t+m-k} - \sum_{k=0}^{m} a_{m-1\,m-k}\,x_{t+k} \quad . \tag{5.1.12b}$$

Since b_{mt} and b'_{mt} are independent of a_{mm},

$$\frac{\partial}{\partial a_{mm}} \Pi_m = 0 \quad . \tag{5.1.13}$$

Thus Π_m is a minimum with respect to a_{mm}; this yields

$$a_{mm} = \frac{2\sum_{t=1}^{N-m} b_{mt}\, b'_{mt}}{\sum_{t=1}^{N-m} \left(b_{mt}^2 + b'^2_{mt} \right)} \quad . \tag{5.1.14}$$

Useful recursion formulas for the arrays b_{mt} and b'_{mt} are derived by means of (5.1.10) and (5.1.12):

$$b_{mt} = b_{m-1\,t} - a_{m-1\,m-1}\, b'_{m-1\,t} \quad , \tag{5.1.15a}$$

$$b'_{mt} = b'_{m-1\,t+1} - a_{m-1\,m-1}\, b_{m-1\,t+1} \quad . \tag{5.1.15b}$$

It is seen that the arrays b_{mt} and b'_{mt} are constructed from the arrays b_{m-1t} and b'_{m-1t} by a simple linear operation. The starting values are

$$b_{ot} = b'_{ot} = x_t$$

for $t=1,2,\ldots,N$. But since these arrays are not used in the calculation procedure, the values for $m=1$ are used instead

$$b_{1t} = x_t \text{ and } b'_{1t} = x_{t+1} \ . \tag{5.1.16}$$

By inserting (5.1.10) into (5.1.7), the recursive formula for P_m is found to be

$$P_m = P_{m-1}\,(1-a^2_{mm}) \ . \tag{5.1.17}$$

5.2 Akaike Final Prediction Error

The number M of filter coefficients required to optimize the results obtained from MEM can be determined by two methods.

One method is based on a window-closing technique similar to the one described by Jenkins and Watts (1968) for the variance spectrum analysis method.

A superior method has been devised by Akaike (1969a, 1969b, 1970). This method, now called the Akaike Final Prediction Error (FPE), has been suggested for use with the ACV method (Akaike, 1971) and with multivariate time series (Fryer et al., 1975), as well as for MEM.

Since the emergence of the Akaike FPE, many authors have incorporated it into the MEM (Akaike, 1969a, 1969b, 1970; Gersh and Sharpe, 1973; Jones, 1973; Ulrych and Bishop, 1975).

The calculation of the Akaike FPE is readily obtained by using

$$(FPE)_M = \frac{N+M+1}{N-M-1} S_M^2 \qquad (5.2.1)$$

where N is the number of data values analysed, M is the number of filter coefficients, and S_M^2 is the residual sum of squares given by

$$S_M^2 = \frac{1}{N} \sum_{n=1}^{N} \left(x_n - \sum_{m=1}^{M} a_{mM} x_{n-m} \right)^2 \qquad (5.2.2)$$

in which a_{mM} are the filter coefficients, x_n are the data values with the data mean removed, and $x_n=0$ for $n \leq 0$.

The objective is to fit the autoregressive model of order M by the least-squares method; this requires the mean square of residuals (5.2.2) to be minimized with respect to $\{a_{mM}; m=1,2,\ldots,M\}$. It is generally observed that S_M^2

decreases and $(N+M+1)/(N-M-1)$ increases as the value of M is increased. Thus the FPE tends to be too large when a large value of M is used. The minimum value of the FPE provides a balance between the order of the autoregressive model and the size of the mean-square prediction error.

An estimate $(FPE)_M$ of the autoregressive model is calculated by (5.2.1). An initial value is calculated using

$$(FPE)_0 = \frac{N+1}{N-1} P_0 \qquad (5.2.3)$$

where P_0 is given by (5.1.8).

For the purpose of comparison of the magnitude of $(FPE)_M$ with $(FPE)_0$, the relative value

$$(RFPE)_M = \frac{(FPE)_M}{(FPE)_0} \qquad (5.2.4)$$

can be used.

5.3 Programs and Data Manipulation

Two subroutines were used to calculate the power spectrum using MEM; they are BURGME and SMSQR. Both are listed in Appendix B.

The data were standardized before being passed on to the subroutine BURGME using (4.3.1) for i=1,2,....,N, where N is the total number of data values.

The subroutine BURGME combines the theory of the MEM with the Akaike FPE. The theory and the flow chart, presented in Andersen's paper, were used extensively in writing the subroutine.

The Akaike FPE has been incorporated as an option such

that the minimum value of $(FPE)_M$ can be automatically found and the associated filter coefficients retained. However, the number of filter coefficients can be specified, bypassing the calculation of the FPE.

An expansion factor has also been incorporated into the program in order to expand the lower end of the spectrum for better definition of peaks. If Q is this expansion factor, then the printout covers the range of frequency from f=0 to $f=f_N/Q$ where f_N is the Nyquist frequency. In this manner, there is no aliasing of values in the range $f=f_N/Q,\ldots,f_N$ onto the range $f=0,\ldots,f_N/Q$; aliasing of frequencies beyond f_N must nevertheless be investigated; this will be done in Chapters 7 and 8.

The subroutine SMSQR calculates the residual sum of squares S_M^2 given by (5.2.2) and required by (5.2.1) if Akaike's FPE is used to obtain the optimum number of filter coefficients.

Sample print-outs are shown in Tables B.18 and B.19 in Appendix B.

CHAPTER 6

AUXILIARY PROGRAMS

6.1 Main and Plotting

The main program (MAIN) links together all the subroutines mentioned in Chapters 4 and 5. It also allows for plotting the raw data and the power spectra from both the FFT and MEM analyses.

The plotting is accomplished with subroutine PLTG which is a slightly modified version of a program available in the Computing Services Program Library of the University of Alberta. The subroutine PLTG is not included in Appendix B as it is too lengthy and could prove difficult to implement on other computer systems.

The subroutines PLTDAT, PLTFFT and PLTMEM provide an interface between MAIN, FASTFT, BURGME, and PLTG. These three interface subroutines isolate most of the commands required when producing a plot and provide an easy method for inserting a different plotting subroutine.

If no plotting facilities are available, the program need only be slightly altered by deleting the subroutines PLTDAT, PLTFFT and PLTMEM and by removing all CALL statements to these three subroutines from MAIN. The three subroutines are listed in Appendix B.

6.2 Mean and Variance

The subroutine VARSD is used to calculate the mean (μ), variance (σ^2) and standard deviation (σ) of the data. It is called once in MAIN in order to obtain these three values from the raw data. It is also called in FASTFT in order to obtain a new set of values from the expanded data set.

The mean is calculated using

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{6.2.1}$$

and the variance is calculated using

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} (x_i)^2 - \mu^2 \quad . \tag{6.2.2}$$

Statistically, the values μ and σ^2 are known as biased estimators of the true population values.

A listing of VARSD can be seen in Appendix B.

6.3 Notch Filter

It was anticipated that some method would be required to eliminate the very strong annual cycle when analysing a full set of temperature data at one-month intervals.

The filter designed for this purpose is called a notch filter; it is also referred to as a rejection filter (Kanasewich, 1975) or a recursion filter in the z-plane (Shanks, 1967).

The following theory, as described by Kanasewich (1975) and Shanks (1967), shows how the notch filter is used to remove the annual cycle. The program, as written (see

Appendix B), can be used to eliminate any frequency.

Consider a unit circle in the imaginary plane. The positive imaginary plane progresses from f=0 to $f=f_N$, the Nyquist frequency.

The general form of the filter equation is given by

$$F(z) = \frac{(z - z_1)(z - z_2)}{(z - z_3)(z - z_4)} \tag{6.3.1}$$

where z_1 and z_2 are the zeroes and z_3 and z_4 are the poles of the filter.

The zeroes, or discontinuities, of the filter occur at

$$z = \cos\theta \pm i \sin\theta$$

where θ is the position on the unit circle of the unwanted frequency. Thus, for f_a, the frequency of the annual cycle, the value of θ is determined by

$$\theta = \frac{f_a}{f_N} \cdot 180° = \frac{1/12}{.5} \cdot 180° = 30°$$

assuming the data interval to be one month.

The zeroes of the filter occur at

$$z_1 = 0.8660 + i0.5000 \tag{6.3.2a}$$

and

$$z_2 = 0.8660 - i0.5000 \quad . \tag{6.3.2b}$$

However, when the poles of the filter coincide with the zeroes on the unit circle, the response is very poor. It is necessary to place the poles near, but not on, the unit circle, say at $|z| = 1.001$. Thus the poles are located at

$$z_3 = 0.8669 + i0.5005 \tag{6.3.3a}$$

and

$$z_4 = 0.8669 - i0.5005 \quad . \tag{6.3.3b}$$

Substituting (6.3.2) and (6.3.3) into (6.3.1), the

filter equation becomes

$$F(z) = \frac{0.9980(z^2 - 1.7320z + 1)}{1 - 1.7303z + 0.9980z^2} \tag{6.3.4}$$

which incorporates a static gain factor to insure a gain of unity at the Nyquist frequency.

Next, the filter is applied to the data using

$$Y(z) = F(z)\ X(z) \tag{6.3.5}$$

where X are the raw data and Y are the filtered data. The recursion equation is obtained by substituting (6.3.4) into (6.3.5), by multiplying both sides of (6.3.5) by the denominator and by rearranging to obtain

$$\begin{aligned} Y(z) &= 0.9980(1-1.7320z+z^2)X(z) \\ &\quad + zY(z)(1.7303-0.9980z) . \end{aligned} \tag{6.3.6}$$

Whenever z occurs in (6.3.6), the input or output is delayed by one unit and similarly z^2 causes a delay of two units. Thus, the recursion equation is

$$\begin{aligned} y_n &= 0.9980\ (\ x_n - 1.7320x_{n-1} + x_{n-2}\ ) \\ &\quad + 1.7303y_{n-1} - 0.9980y_{n-2} \end{aligned} \tag{6.3.7}$$

where x_n is the data input and y_n is the output from the filter.

The amplitude response of this filter is shown in Figure 6.1 in which 120 frequency values are assumed to lie in the interval $0 \le f \le f_N$.

The recursion equation (6.3.7) is programmed to filter the data in a cascade fashion. A cascade consists of passing the filter twice over the data set, once in a forward direction from x_1 to x_N and once in a backward direction

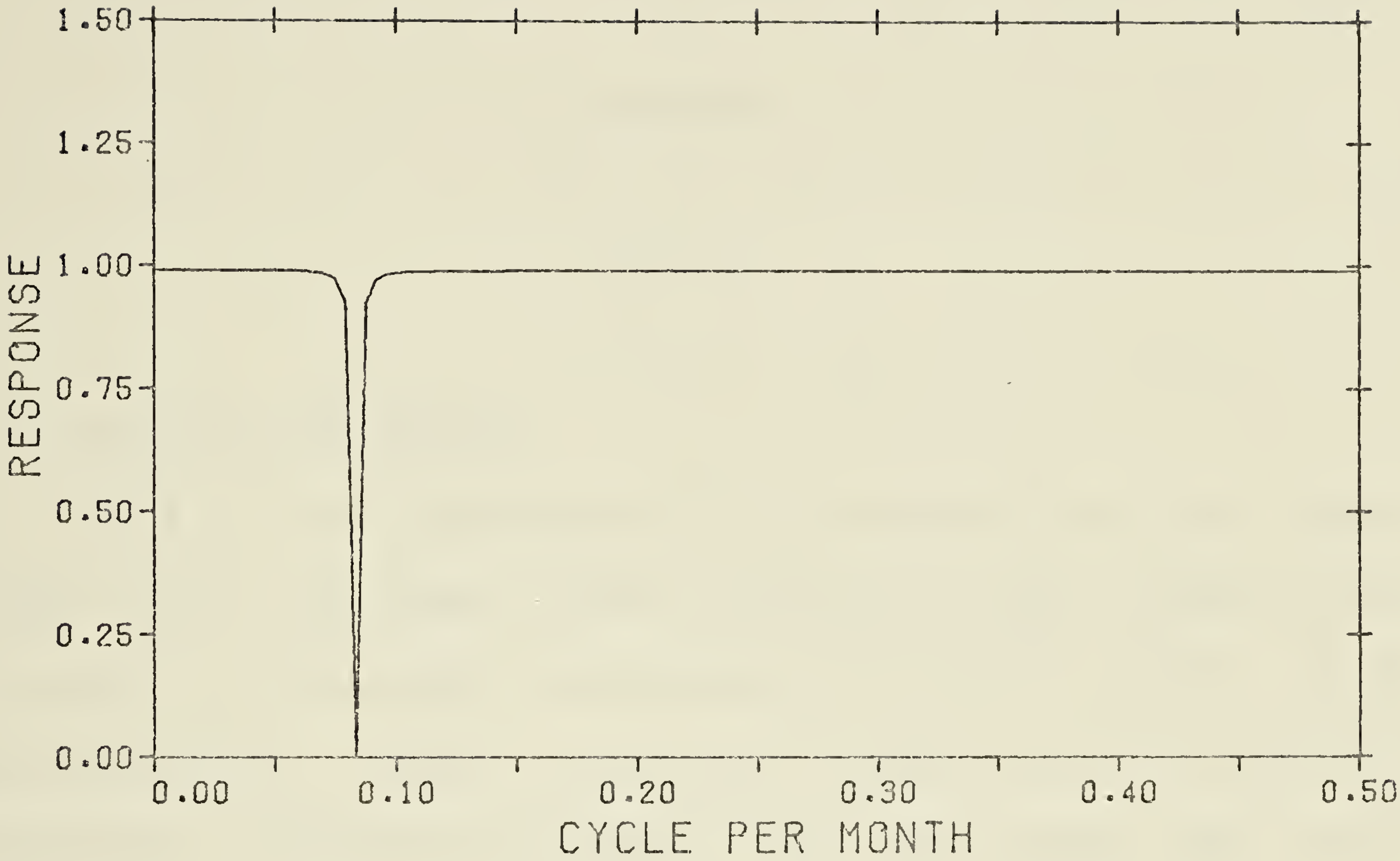

Figure 6.1. Amplitude response of the notch filter for the annual cycle.

from x_N to x_1. The cascade prevents a phase shift in the data. Also, a sufficient number of zeroes must be added to the ends of the input data to allow the output from the initial pass of the filter to become significantly small.

The program for the notch filter is separate from the spectral program.

CHAPTER 7

RESULTS

7.1 Program Performance

Part of the development of the programs involved tests which were performed using synthetic data. These data consisted of different combinations of five to seven exact periodicities. Every generated wave had a specific frequency, amplitude and phase angle. The waves had small amplitudes at low frequencies and larger amplitudes at higher frequencies. The periods ranged from 100 to 2 "years". The length of data used was equivalent to approximately 70 years of real data. Thus, when using the modified periodogram, zeroes were added to the generated data as if real data of finite length were being used. These generated data were analysed using the modified periodogram method and MEM.

The modified periodogram performed quite well and was able to analyze the periodicities provided. Periods greater than approximately one-third the length of data were not resolved as accurately as the shorter periods. Periods approximately equal to or greater than the length of data were detected at the first frequency value above zero, but no values of frequency, amplitude or phase could be

determined with precision. The values of the spectral peaks exceeded the 99.9% significance level for the resolvable waves present in the synthetic data (significance levels are discussed in the next section). The spectral peaks were analyzed at the output frequencies which were closest to the frequencies specified in generating the data. The weighted frequency proved useful in this regard because it provided interpolated values of frequencies which were better estimates of the actual values. Similarly, the smoothed phase, obtained using a three-point Daniell filter on the real and imaginary parts of the Fourier transforms, also provided better estimates of the phase angles than the unsmoothed phase angles.

Tapering was performed on the data in order to assess the effects of truncating the data set. No improvements were observed in the resolution properties of the periodogram. All periods within the range of detectable frequencies were revealed with equal or slightly superior results when no tapering was applied to the ends of the data.

The Maximum Entropy Method did not prove to be as accurate or as convenient as the modified periodogram. Many observations made about MEM in this study were also discussed by Chen and Stegen (1974).

The spectral output of MEM was more difficult to interpret. It required considerable time and care when determining the power resolved for a certain frequency, since it was not the value of the peak which provided an

estimate of the power but rather the area under the curve. For example, the maximum single value of variance attributed to two periods of equal amplitude, among a combination of five generated periods, differed by a factor of twenty in the same analysis. Thus, confidence limits could not be applied as easily to MEM results as was possible with the modified periodogram results. At this time, the true meaning of the numerical values calculated by MEM is unknown. The value of a spectral peak is subject to wild fluctuations when slightly different data sets are used. The periodogram method was not subject to such wild fluctuations and the maximum single value of variance attributed to periods of equal amplitude differed by less than a factor of two. MEM, unlike the periodogram method, must be calibrated using test data before being used on real data.

The resolution properties of MEM were highly dependent on the number of filter coefficients used, especially if MEM was used to determine the value of a very low frequency. The number of coefficients required was usually higher than the number determined by the Akaike FPE. Thus, MEM required a process similar to the determination of the optimum lag value for the ACV method.

Also, the spectral peaks analyzed were not necessarily at the proper frequency. Chen and Stegen (1974) found frequency shifts which resulted from a combination of the initial phase and the data length. At this stage, MEM can only provide an indication of low frequencies present in the

data. However, a recent paper by Ulrych and Clayton (1976) discusses a method by which the frequency shift may be reduced considerably. This paper was brought to the attention of the author by Dr. T.J. Ulrych during a private conversation. Unfortunately, this conversation took place at a very late stage of this study and no time was available to make the modifications necessary to improve the method.

MEM was found to be quite costly in computer time, especially when using long data sets. The time required when calculating the Akaike FPE was also significant and was mainly due to the calculation of S_M^2 using (5.2.2). A modification, obtained more by accident than by design, was discussed with Dr. Ulrych. He outlined the justification for replacing the value of S_M^2 obtained from (5.2.2) by the latest value of P_M obtained by (5.1.17). This substitution is justified since both S_M^2 and P_M are measures of the variance of the error between the calculated set of filter coefficients and the ideal set of filter coefficients. Substituting P_M for the calculation of S_M^2 would reduce the computational time required.

The notch filter was found to be quite effective in reducing or eliminating an unwanted frequency. A "before and after" example of the effect of the filter is shown in Figures 7.1 and 7.2. The upward shift of the plot in Figure 7.2 results from the redistribution of that portion of the normalized power which has been released by eliminating the annual cycle.

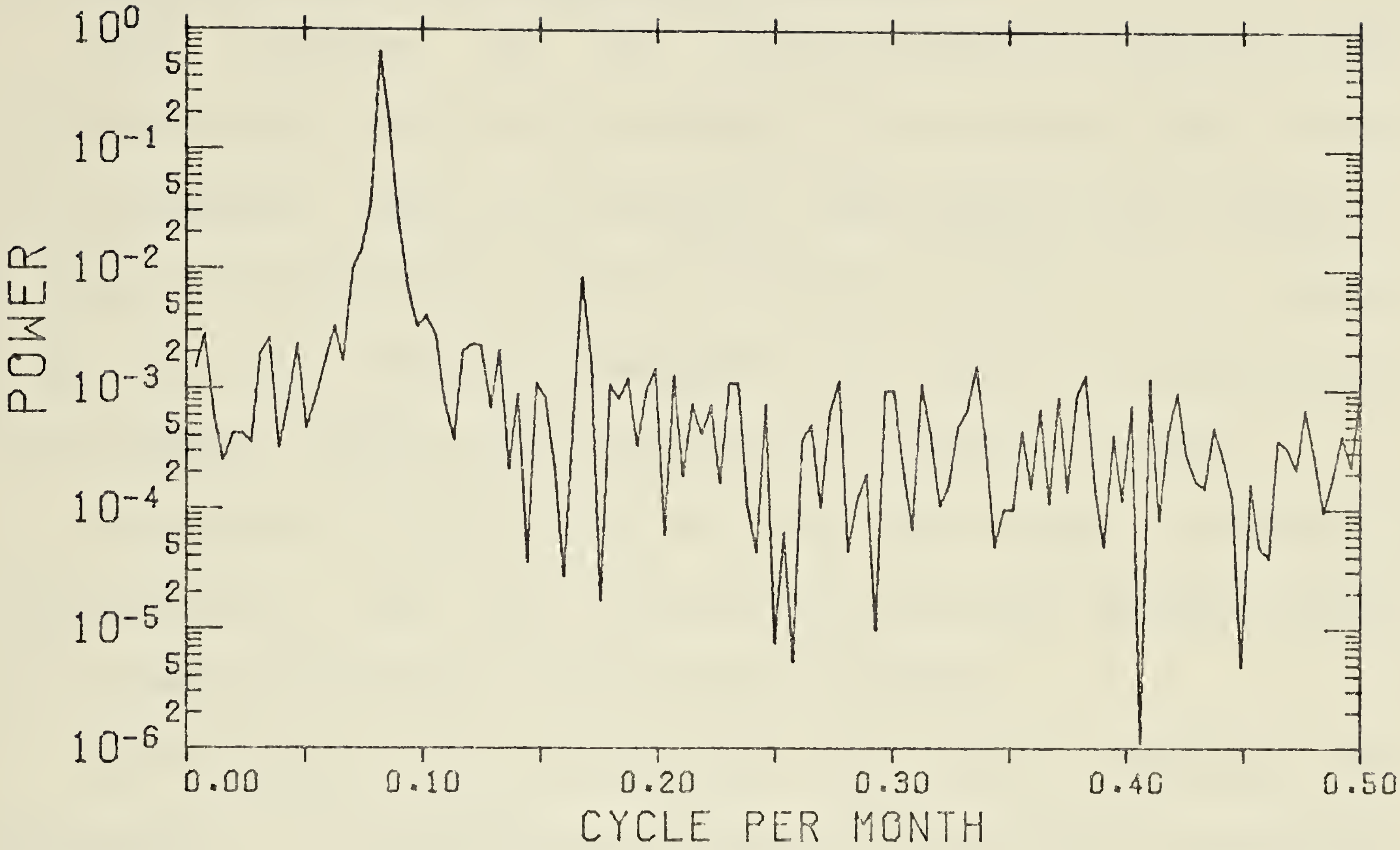

Figure 7.1. Periodogram analysis of FVR monthly temperatures with no filtering applied to the annual cycle.

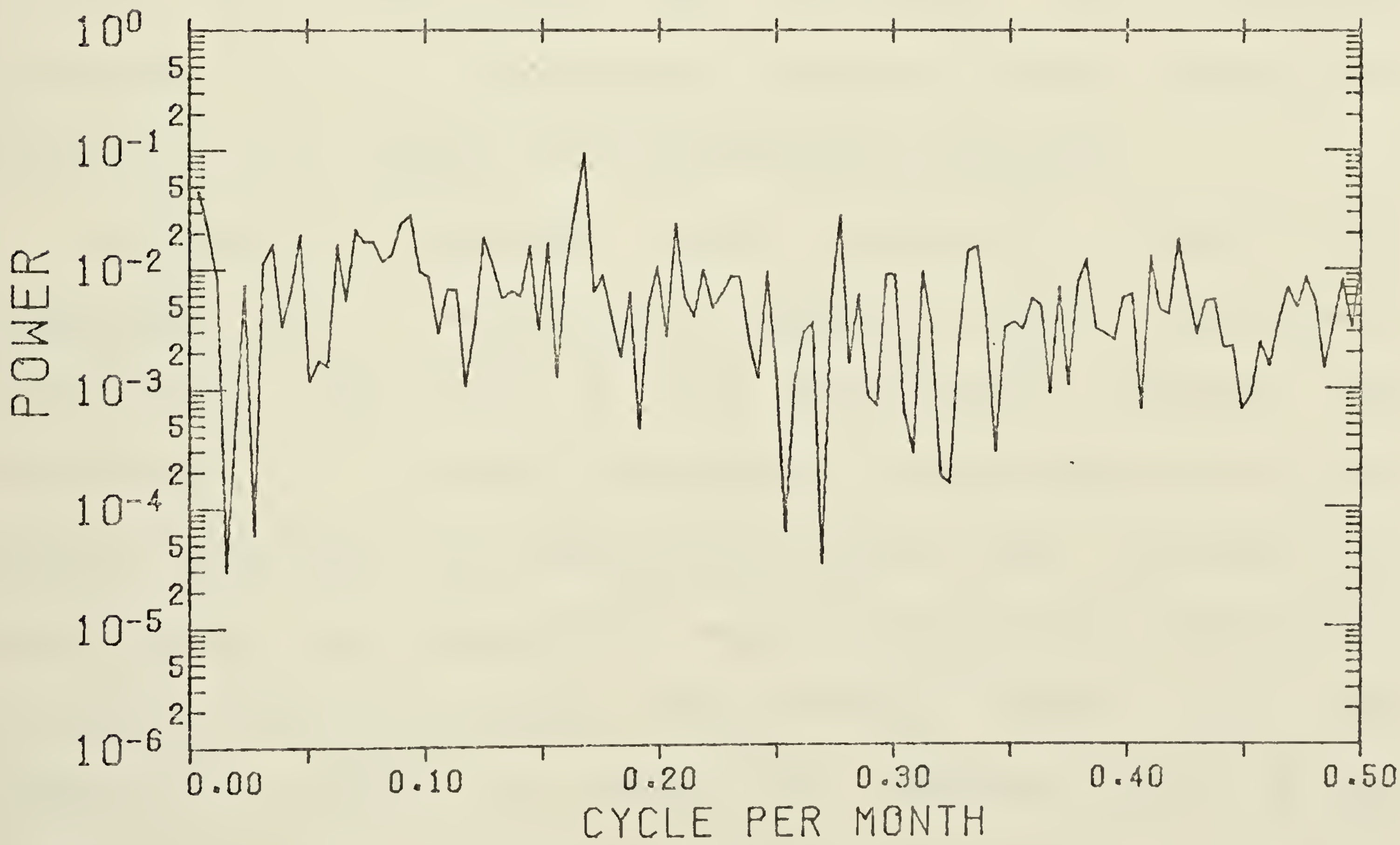

Figure 7.2. Periodogram analysis of FVR monthly temperatures with the annual cycle removed.

7.2 Data Analysis and Evaluation Procedure

The results of the climatological analysis are discussed under four main headings. This section deals with the procedure used to analyze and evaluate the results. Section 7.3 deals with results affecting Alberta as a whole based on all stations analyzed. The remaining sections of this chapter discuss each station on a regional basis.

The primary method used was the periodogram analysis. Unfortunately, MEM was used very little for reasons which have been detailed in the previous section.

The procedure used to analyze the data consisted of performing a periodogram analysis on various data groupings as outlined in Section 1.3. The power spectra obtained were plotted on a Calcomp plotter. Both smoothed and unsmoothed spectra were calculated. The smoothing was normally performed with a three-point Daniell filter. Nearly 600 graphs of power spectra were eventually produced.

Confidence intervals were calculated using the assumption of a white-noise spectrum and a Chi-square distribution. The value of the white-noise variance was dependent on the number of spectral lines analyzed and was easy to determine. The confidence limits were obtained by multiplying the white-noise value by $\chi^2_{1-\alpha}/\nu$ and χ^2_{α}/ν for the upper and lower limits, respectively, where ν is the appropriate number of degrees of freedom and α is the probability level selected. The degrees of freedom for the periodogram analysis were determined by using

$$\nu = \left(\frac{N-2}{2^{n-1}}\right) D \qquad (7.1.1)$$

where N is the number of independent data values used in the analysis, 2^{n-1} is the number of spectral lines obtained when the N data values are supplemented by 2^n-N zeroes (see Section 4.3), and D is the width of the Daniell filter used on the periodogram (D=1 when the periodogram is unsmoothed). The number of data values is reduced by two degrees of freedom as a result of using the data to calculate the mean and variance values.

The degrees of freedom for the unsmoothed periodogram varied between one and two. If the calculated degrees of freedom were 1.5, an erroneous result could have been obtained by using ν=1 (over-restricting) or ν=2 (under-restricting) since the distribution of the Chi-square function varies considerably and non-linearly between these two values of ν. For this reason, graphs were plotted of the Chi-square function in order to obtain more accurate estimates of the confidence intervals for both unsmoothed and smoothed spectra.

The 95% and 5% confidence limits and the white-noise spectrum were drawn on the graphs as shown in Figure 7.3. All spectral lines exceeding these limits were noted along with the significance level reached. The levels considered were 0.5%, 1%, 2.5%, 5%, 95%, 97.5%, 99%, 99.5%, and 99.9%. The spectrum illustrated in Figure 7.3 is typical of the spectra obtained in this study and it demonstrates the

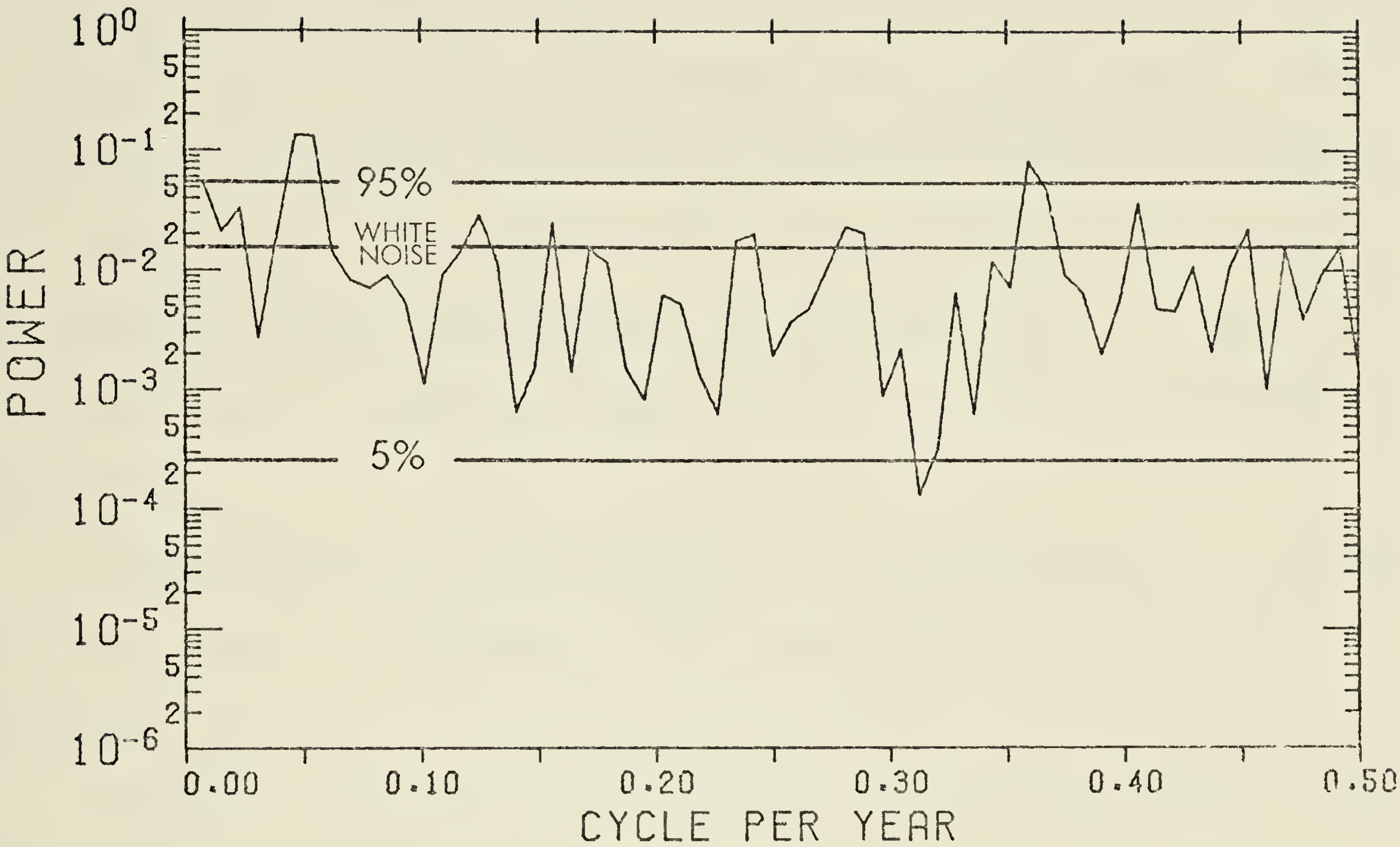

Figure 7.3. Periodogram analysis of YXH winter precipitation showing a white-noise spectrum and a 90% confidence interval.

validity of assuming a white-noise model.

The analysis for all months and for each month of the calendar year was accomplished using the data as collected. The temperature and precipitation data were averaged and summed respectively over the appropriate time interval to obtain the annual and seasonal values. The groupings were made as follows:

1- Annual - January to December.

2- Winter - December, January, February.

3- Spring - March, April, May.

4- Summer - June, July, August.

5- Fall - September, October, November.

These groupings, as opposed to the full sets of monthly values, were chosen because it was felt that they would eliminate the necessity of removing the annual cycle which generally accounted for 80% or more of the data variance. The notch filter proved capable of eliminating this period, however, possible side-effects of removing such a tremendous portion of the variance might have influenced the remaining results.

Beaverlodge was chosen for an in-depth analysis of some of the results obtained. This analysis is discussed in Section 7.3.

Periods of 32 years or greater were considered to be very doubtful because, in most cases, only 64 spectral lines were analyzed. An example of such an analysis is shown in Table B.15. The first five values of frequency output correspond to periods of ∞, 128, 64, 42.7, and 32 years. A significant peak with a period of 32 years or more was viewed as possible but not necessarily definitive evidence of long periodicities present in the data. Such significant periods are included in Section 7.3. However, a discussion of their validity is reserved for Section 7.4.

In addition it is often difficult to decide whether a wave is significant by itself or is a harmonic of some other wave; for example, significant periods of 8 and 4 years could easily result from the analysis of a single 8-year wave pattern where both waves are needed to describe accurately the wave. The identification of such a harmonic

wave would simplify the task of finding the causes of significant periods.

It must be remembered that the analyses of annual, seasonal, and individual month variations for BEA and MAN provided only 32 spectral estimates between zero and the Nyquist frequency while 64 estimates were obtained for all other stations. Also, 5% of these estimates were expected to exceed the 95% confidence limit. Therefore, an analysis with three estimates exceeding the 95% significance level out of 64 spectral estimates did not necessarily provide meaningful results, as will be shown in the remaining sections of this chapter.

7.3 Alberta as a Unit

A province-wide comparison was limited mostly to results obtained from annual and seasonal values of temperature and precipitation. This limitation was due to the volume of results generated and the time restrictions involved. A more detailed analysis of individual months is discussed in the last three sections of this chapter.

The "spectral gaps" or estimates which are below the 5% confidence limit are not discussed at this time. Instead, attention is focused on frequencies or groups of frequencies that appear to account for relatively large amounts of total variance.

Tables 7.1 to 7.5 show the results obtained for the analysis of annual and seasonal values. Only significant

Table 7.1. Periods (years) of significant peaks and significance levels (P,%) for annual values.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
FVR	128.	64.0 16.0	14.2			128. 5.33
BEA	16.0+	64.0				7.11
RAN		none			none	
YXD		none			10.7	2.25
LAC			2.67 2.61		9.85	9.14
LTH		2.67				12.8 4.27
YXH	64.0+	128.	4.92		18.3	25.6 14.2 2.37
MAN			21.3			64.0

+ indicates that the peak exceeded the 99.5% level.

peaks (≥95%), obtained from the unsmoothed periodogram analyses, are listed in these tables. The reasons for not using the smoothed periodogram results are discussed in Section 7.5.

Few results for any one particular station exceeded the expected number of significant peaks. One such case is in the winter precipitation of YXH from which four significant peaks were resolved. However, periods 21.3 and 18.3 are adjacent peaks (see Figure 7.3) which are likely to indicate a single period of approximately 19.7 years. This then reduces the number of peaks to the expected number although

Table 7.2. Periods (years) of significant peaks and significance levels (P,%) for winter values.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
FVR			16.0	128. 64.0++		42.7 4.27
BEA		16.0	3.20 2.21		none	
RAN			3.28 2.25			2.37
YXD	3.28		10.7 2.21			3.20
LAC		none				4.74
LTH		6.74 3.28				64.0
YXH		3.28	2.46	21.3+ 18.3+	2.78	128.
MAN		3.20	16.0 7.11 3.39	64.0+		

\+ indicates that the peak exceeded the 99.5% level,
++ indicates that the peak exceeded the 99.9% level.

the fact remains that these adjacent peaks are both significant at the 99.5% level and that together they account for 26.6% of the variance of the data. This result is considered further in Section 7.6.

Three results are the most prominent when all stations are examined. All three results come from the temperature analyses. They consist of a 3.2- to 3.3-year period in the winter at six stations, a 3.8- to 4-year period in the spring at five stations, and a 4.9- to 5.1-year period in

Table 7.3. Periods (years) of significant peaks and significance levels (P,%) for spring values.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
FVR		128.	16.0 14.2			6.74
BEA	16.0+				none	
RAN			3.88 2.42		128.	64.0
YXD			5.12 4.00 2.61		none	
LAC			3.88		none	
LTH		3.88			none	
YXH			128. 64.0 4.00 2.61			25.6 18.3 6.74
MAN		2.46				10.7

+ indicates that the peak exceeded the 99.5% level.

the fall at all stations.

The most prominent result is the approximate five-year period in the fall temperature values. All stations have a significant peak at 4.92 years and five of these stations have a second significant peak at an adjacent period such that the true period can be thought to lie somewhere between the two peaks. In the fall values, the amplitudes of the significant peaks at and adjacent to the 4.92-year peak account for 10 to 30% of the variance of the data used. This result is as baffling as it is surprising because it

Table 7.4. Periods (years) of significant peaks and significance levels (P,%) for summer values.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
FVR	32.0	128. 42.7	8.00			3.39 2.67
BEA		2.37	16.0 2.13			7.11
RAN		42.7	21.3		none	
YXD	128.++		32.0	10.7	12.8	
LAC			25.6			3.80
LTH		12.8	42.7		12.8	
YXH	64.0++		128. 42.7 12.8 4.00	2.37+		12.8
MAN		none			none	

\+ indicates that the peak exceeded the 99.5% level,
++ indicates that the peak exceeded the 99.9% level.

provides possible support for the results obtained by Georgiades (1977) while having no known physical cause.

This five-year period was investigated as to possible sources other than a true periodicity present in the data. This investigation was carried out using fall temperatures for BEA. A plot of the fall temperatures is shown in Figure 7.4. The periodogram analysis of the raw data is shown in Figure 7.5. An 11-point Tukey filter was passed over the raw data; this reduced the number of data values from 61 to 53. The filtered data are shown in Figure 7.6 and the

Table 7.5. Periods (years) of significant peaks and significance levels (P,%) for fall values.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
FVR		5.12	4.92		none	
BEA	4.92	6.40	2.56		32.0	3.05
RAN		5.12 4.92				2.84 2.78
YXD	4.92+					4.57
LAC		5.12 4.92			10.7	6.40 2.51
LTH		5.12 4.92	7.53		none	
YXH	4.92+		7.53			16.0
MAN	4.92		4.57			9.14

\+ indicates that the peak exceeded the 99.5% level.

periodogram analysis of these data is shown in Figure 7.7.

Comparing Figures 7.5 and 7.7 it will be seen that the power of the low frequencies was reduced and the power of most higher frequencies was increased slightly as a result of the redistribution of the total power. Thus leakage of power from the low frequencies did not appear to contribute to the power of the five-year period.

The next problem was to determine if aliasing had taken place. The power spectra of daily and monthly temperature values were not included in this thesis due to the size of the figure required for adequate display. A series of daily mean temperature values was readily available for BEA. An

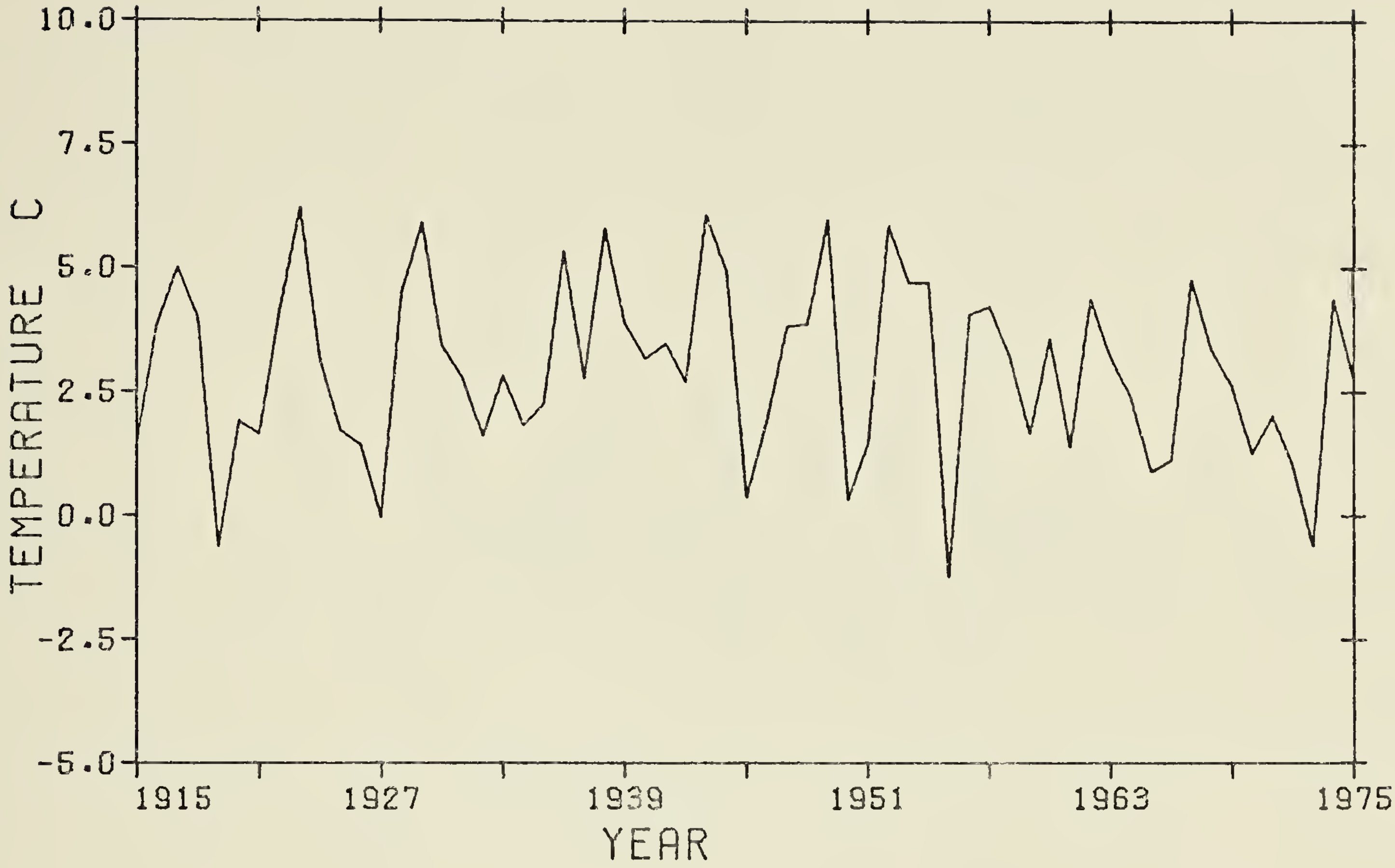

Figure 7.4. BEA fall temperatures.

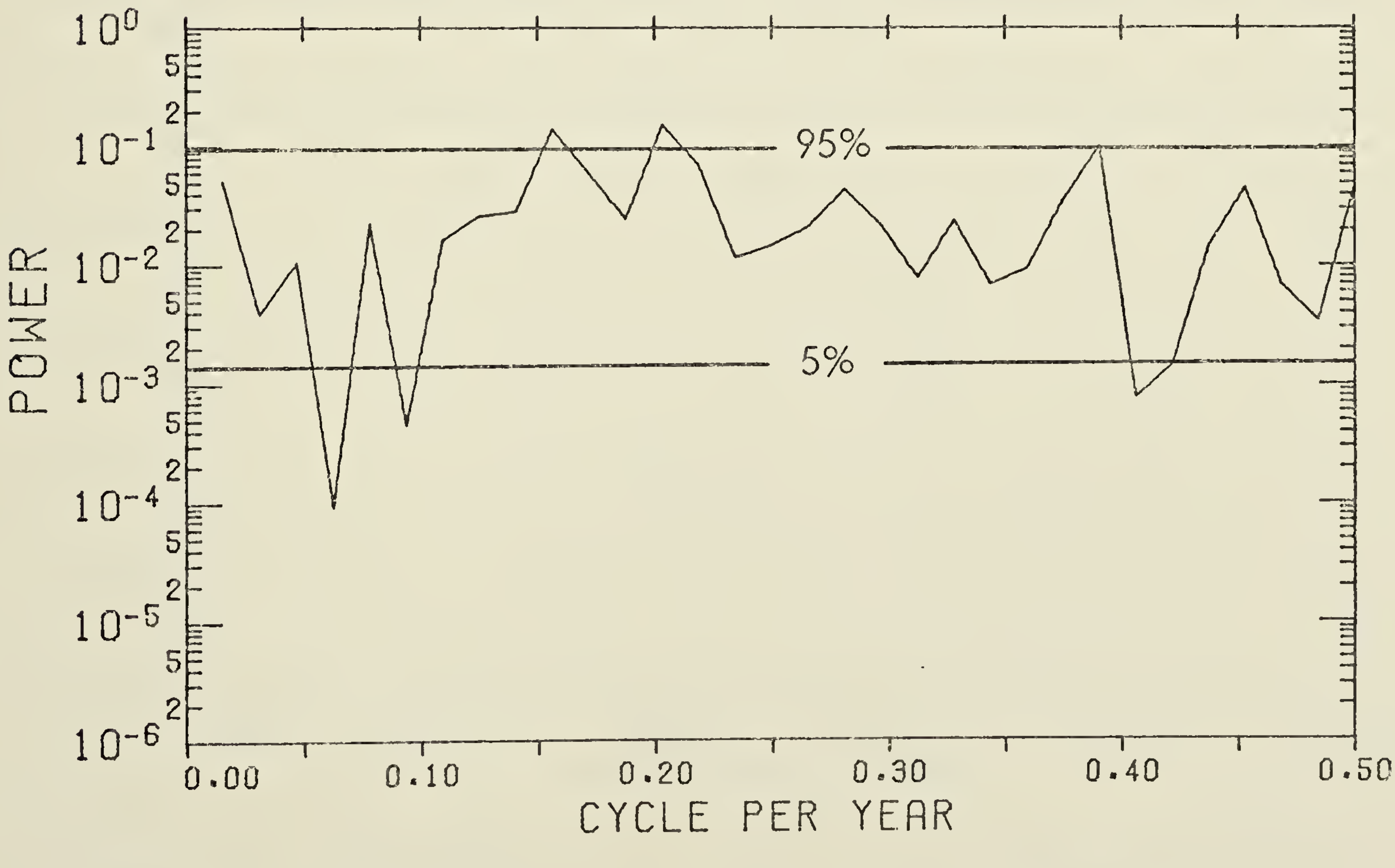

Figure 7.5. Periodogram analysis of BEA fall temperatures.

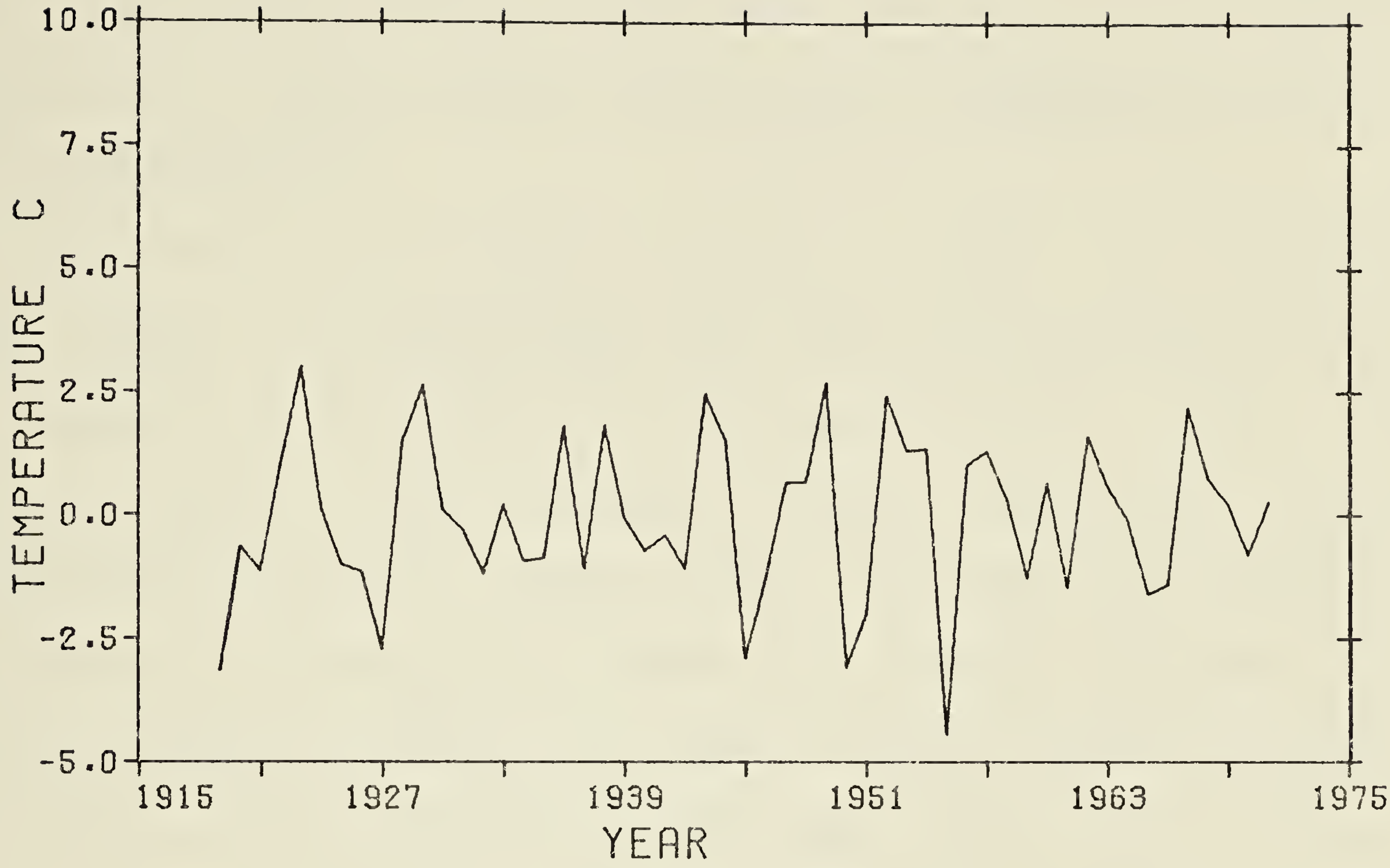

Figure 7.6. Filtered values of BEA fall temperatures.

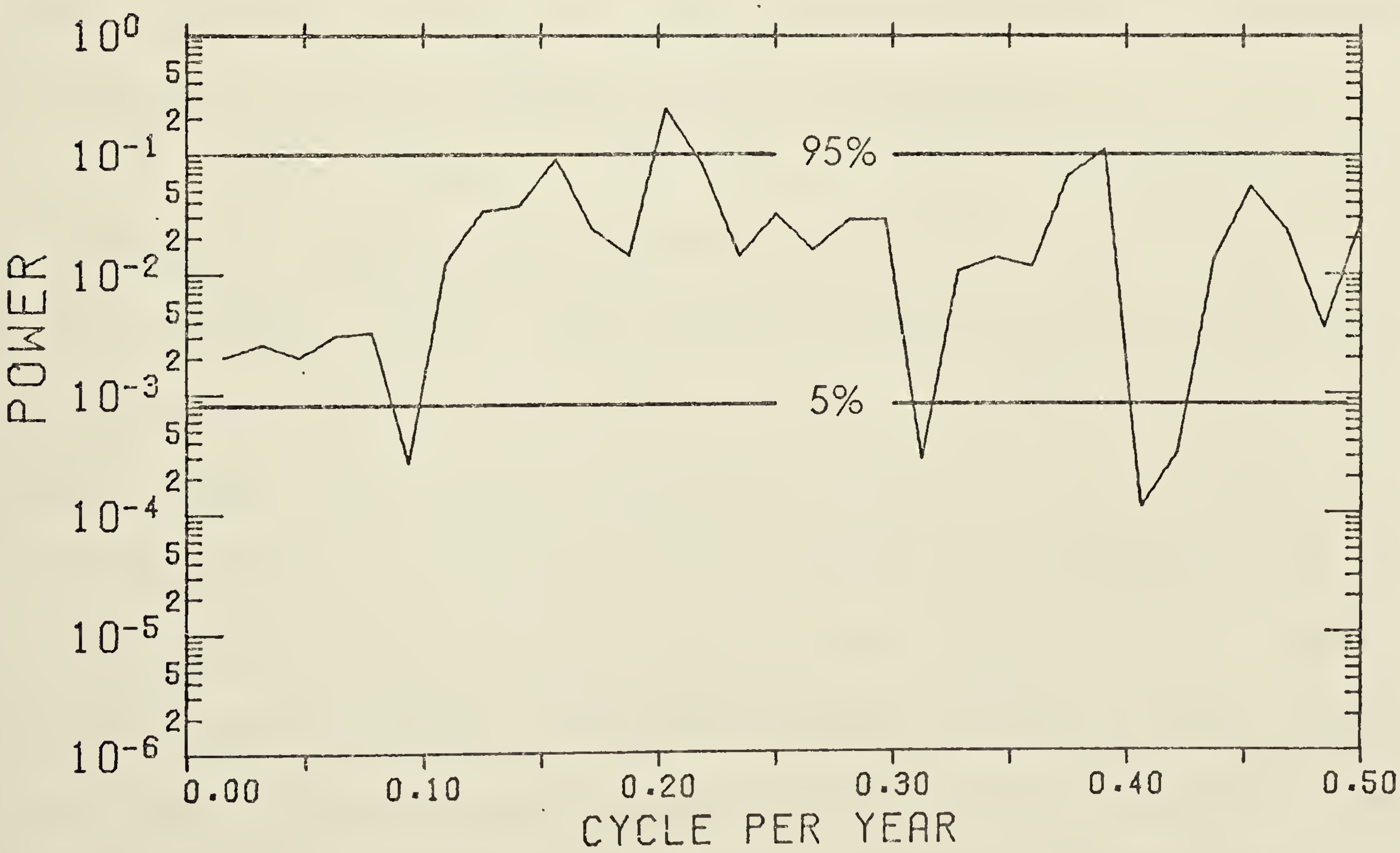

Figure 7.7. Periodogram analysis of the filtered values of BEA fall temperatures.

analysis of four consecutive years, 1950 to 1953, showed that daily values follow a red-noise spectrum. The annual cycle was clearly evident and no other frequency was found to be significant.

The analysis of monthly values, in which all monthly mean values of temperatures were used, was performed after the removal of the annual and semi-annual cycles using the notch filter. This analysis showed significant peaks associated with 16.7-, 1.82-, 0.30-, and 0.208-year periods. The last three periods are beyond the Nyquist frequency of 0.5 as shown in Figure 7.5. Using a linear folding method, the three periods would be aliased onto the 2.22-, 3.28-, and 5.0-year periods. However, the variance observed at each of the three highest frequencies was less than 1% of the total variance of the data. All these factors are discussed in the next chapter before drawing conclusions.

At this time, a few more facts concerning fall temperature analyses are presented. Of the eight stations, three records (FVR, LAC and LTH) began in 1908. The phase angles computed were 43°, 2°, and 1°, respectively. The phase angle for YXD, beginning in 1881, was 51°, and for YXH, beginning in 1884, was -82°; using an approximate value of 72° for the phase change each year based on an exact period of five years, the phase angle for YXH in 1881 would have been 62°. The phase angles obtained by adjusting the values for BEA and RAN from 1915 and 1905 to 1908 were 60° and 38°, respectively. In order to compare the phase angle

of two stations, a correction of 72° per year was applied to the derived phase shift. This correction is not likely to be exactly 72° such that 30- to 40-year corrections could not easily be applied in order to compare all stations. The similarity of the resolved phase angles is nonetheless striking.

The amplitude of the five-year period was approximately 1C for most stations. The values which differed the most from 1C were 0.78C at YXD, 0.84C at YXH, and 1.16C at MAN. The data variance was smallest at MAN with a value of 2.20C^2 such that this period accounted for 30.4% of the variance.

The values obtained from the analyses of individual months also provided some interesting results with respect to the five-year period. Significant peaks associated with this period were found for six stations in the September data, and for five stations in the November data, but no station revealed such a peak in the October data. It was found upon closer examination that, for all stations, the percentage of variance explained in the October data by an approximate five-year period was equivalent to, or less than, the percentage explained by a white-noise spectrum.

The analyses of summer values revealed no single frequency which was as frequently significant as the frequencies revealed for the other seasonal values. Generally, the significant peaks occurred at very low frequencies. Such a result is very difficult to interpret using the periodogram, and, as discussed in Section 7.1, the

MEM analysis would add little information.

Both stations which did not exhibit a 3.2- to 3.3-year period in winter temperatures at the 95% confidence level did have such a period at the 90% confidence level. Similarly all three stations which did not exhibit a 3.8- to 4-year period in spring temperatures did have such a period at the 85 to 90% confidence level. Considerable doubt is cast on the statistical significance of these results by going below the 95% level. However, because so many stations had these results in common, the presence or absence of these periods at the other stations could strengthen or weaken the hypothesis that an approximate 3.2-year period in winter temperatures and an approximate 4-year period in spring temperatures may be found at many stations across the province although the true existence of these periods at the stations with the low confidence level is doubtful.

Closer scrutiny of the individual months used to derive the winter temperature values showed that, for December temperatures, all stations except RAN had significant peaks at the 95% level for a 3.12- to 3.2 year period; that, for January temperatures, only RAN and YXD had such peaks for a 3.28-year period; and that, for February temperatures, six stations had such peaks for a 3.28- to 3.39-year period while the other two stations had peaks for a 3.56- to 3.66-year period.

A similar comparison done for spring temperatures showed that, for March temperatures, only FVR and MAN did

not have significant peaks for a 4-year period; that, for April temperatures, no station had any significant peak close to a 4-year period; and that, for May temperatures, only RAN and YXD had significant peaks for a 4-year period.

7.4 Northern Alberta

Figure A.2 in Appendix A shows the location of all Alberta stations used in this study. BEA and FVR results are discussed separately in this section. All other stations are dicussed in the next two sections. The terms Northern, Central and Southern Alberta have been losely applied and are used here as a convenient way of grouping the stations.

Tables 7.6 to 7.9 present the results obtained for FVR and BEA based on the seasonal, annual and individual monthly values of both temperature and precipitation. Only significant spectral peaks (≥95%) are listed in these tables. All peaks were obtained from the unsmoothed periodogram analyses.

A marked feature found in the analyses of FVR values was the relative absence of spectral lines with values less than the 5% significance level. This result was common to most of the stations analyzed. The stations with the greatest number of such gaps analyzed were BEA, YXD and YXH. This finding cast considerable doubt on the usefulness of significant gaps. Alternatively, the question arose as to the meaning of a significant gap for the purposes of this study, since the amplitude of the wave associated with such

Table 7.6. Periods (years) of significant peaks and significance levels (P,%) for annual and seasonal values at FVR.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
Annual	128.	64.0 16.0	14.2			128. 5.33
Winter			16.0	128. 64.0++		42.7 4.27
Spring		128.	16.0 14.2			6.74
Summer	32.0	128. 42.7	8.00			3.39 2.67
Fall		5.12	4.92		none	

++ indicates that the peak exceeded the 99.9% level.

a gap would be near zero. It is of interest, for some purposes, to know which periods are not present in a time series, but, this is not the case for meteorological time-series prediction. For these reasons, the significant gaps analyzed were not included in the tables presented in this chapter.

Table 7.6 presents the analyses of annual and seasonal values of temperature and precipitation at FVR. Periods of 64 and 128 years appear a number of times in this table as well as in Table 7.7. Such periods have little meaning when analyzed by the periodogram method. MEM was used to obtain an estimate of the possible long period indicated by the periodogram analysis of FVR annual temperature values. The analysis was performed using 22 filter coefficients on 66

Table 7.7. Periods (years) of significant peaks and significance levels (P,%) for monthly values at FVR.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
Jan.		18.3	16.0 2.25			3.39
Feb.			3.66	64.0+	128.	
Mar.		16.0	14.2	14.2		6.40
Apr.		3.05			none	
May		128.	42.7 32.0			5.82
Jun.		32.0	128.		none	
Jul.		42.7 18.3	32.0 5.57			3.39 2.29
Aug.			32.0 8.00		12.8	32.0 14.2
Sep.		none			2.13	
Oct.			18.3		none	
Nov.	2.56	2.61			128.	64.0 5.57
Dec.			3.12	64.0++	128. 42.7	5.12

+ indicates that the peak exceeded the 99.5% level,
++ indicates that the peak exceeded the 99.9% level.

data values. MEM revealed a peak at the 82.5-year period. However, the number of data values used and the initial phase may have shifted this peak from the true value, as described in Section 7.1. An estimate of the true period of this peak might be 82.5±10 years, based on the results

Table 7.8. Periods (years) of significant peaks and significance levels (P,%) for annual and seasonal values at BEA.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
Annual	16.0+	64.0				7.11
Winter		16.0	3.20 2.21		none	
Spring	16.0+				none	
Summer		2.37	16.0 2.13			7.11
Fall	4.92	6.40	2.56		32.0	3.05

+ indicates that the peak exceeded the 99.5% level,

obtained during the analysis of the test data. Such an estimate was of little use in finding true periodicities.

Assuming, for the moment, that an 82.5-year period did exist in the FVR annual temperature values, how real would this period be? Chapter 2 showed that there were no monthly mean temperature values missing in the FVR record. However, a change of location of the observing site was noted in Chapter 3. This change took place in 1936, a few years prior to the mid-point of the data record, and involved a move of nearly 8km to the new site. This move could have easily resulted in a change in the surrounding temperature regime which could be interpreted as a portion of a very long period. The existence of physical differences between the two sites is not known and could only be determined by an on-site investigation.

Table 7.9. Periods (years) of significant peaks and significance levels (P,%) for monthly values at BEA.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
Jan.		16.0			none	
Feb.		3.56				3.56 3.05
Mar.	16.0	10.7	4.00	4.00		64.0 16.0
Apr.		3.05			none	
May			64.0		none	
Jun.			2.13	3.76		
Jul.	16.0					2.06
Aug.		2.37				5.82
Sep.			6.40 4.92			2.46
Oct.			5.33		3.05 2.00	21.3 5.33
Nov.	2.56++			32.0+		6.40
Dec.			3.20 2.21		7.11	8.00

\+ indicates that the peak exceeded the 99.5% level,
++ indicates that the peak exceeded the 99.9% level,

An article recently written by Schaal and Dale (1977) creates more doubt about the existence of long periods in data samples. Briefly, they found that, because of a change in the time period used to describe the "climatological day", the mean March temperature over Indiana acquired a bias of 0.67C which is nearly equal to the amount of

climatic "cooling" observed in the mean March temperature values of Indiana over the last 40 years. Mr. T. Donnelly, Chief of Surface Inspectors at WRO, related that, shortly after the second World War, the World Meteorological Organization standardized the observation times. The end of the climatological day in Canada was moved from 0000GMT to 0600GMT for the AES-controled stations. Mr. Donnelly also advised that no regulations are imposed at privately-operated stations and that the time of observation may vary, presumably as the work load of the observer varies.

In view of these findings, it is impossible to say whether a period of 82.5 years with an amplitude of 0.64C at FVR is real or results from some or all of the factors mentioned above. This uncertainty added to the reluctance to use MEM because apparent waves could have been introduced into the data by one or more obscure factors.

It is interesting to note that the results for BEA, shown in Tables 7.8 and 7.9, contained few very long periods. The site at BEA was moved once in 1958. Simultaneous records were kept at the two sites during the three-year period preceding the move. Carder (1962) referred to this move and stated that the comparison, which he carried out, did not reveal any significant difference in the temperature and precipitation records. Unfortunately, the data collected during these three years were never published and are not available from the station. However, accepting Carder's statement would be equivalent to ignoring

the change of location of the observing site. The only result affected by such an assumption would be the 64-year period found in the annual temperature values. However, too many unknown variables remain and any conclusion associated with this 64-year period could only be called irresponsible.

All further discussion in this chapter deals with significant peaks with periods of less than 32 years.

Very few analyses of FVR or BEA data values showed any "unexpected" results because, as mentioned in Section 7.2, 5% of the analyzed spectral lines were expected to exceed the 95% significance level.

The fairly frequent occurrence of a 16-year peak, mainly in the temperature analyses for FVR and BEA, was investigated but little agreement was found among the phase angles. Regardless of the 99.5% significance level achieved by the 16-year period in the analyses of annual and spring temperature values at BEA (see Table 7.8), there was insufficient evidence available for any positive statement.

The shorter periods, towards the 2-year "Nyquist" period, showed little organization. Too few of these periods, derived from the analysis of individual months, were evident in the results of analyses of seasonal or annual data. There was no pattern evident, either at FVR or BEA, or common to both, other than that discussed in the previous section.

The data values which were listed as missing in Table 2.2, were of little concern because few of the results of

the precipitation analyses were of interest. The two values, missing from the BEA record, were for August 1915 and January 1916; both months were in the first year of observations. The five missing values for the FVR precipitation record were September-October 1911, March-April 1916 and April 1934.

7.5 Central Alberta

Tables 7.10 to 7.15 present the results obtained for RAN, YXD and LAC based on the seasonal, annual and individual monthly values of both temperature and precipitation. Only significant spectral peaks (≥95%), obtained from the unsmoothed periodogram analyses, are listed in these tables.

The three stations considered in this section exhibit patterns similar to those which have already been discussed in Section 7.3. Periods of the order of 2.57 years appear in a number of places but with insufficient regularity and significance to indicate extraordinary circumstances. There are, however, one or two periods worthy of mention.

First, consider the 18.3- to 21.3-year period analyzed from the July temperatures of both RAN and LAC. The 18.3-year period was also apparent in the January temperatures of these two stations. Of the four occurrences of this peak, only the July peaks for RAN exceeded the expected number of three significant spectral peaks. The January peak for LAC lacked credibility because it was the only spectral line

Table 7.10. Periods (years) of significant peaks and significance levels (P,%) for annual and seasonal values at RAN.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
Annual		none			none	
Winter			3.28 2.25			2.37
Spring			3.88 2.42		128.	64.0
Summer		42.7	21.3		none	
Fall		5.12 4.92				2.84 2.78

which exceeded the 95% significance level. A few observations are presented concerning the July peaks.

The weighted frequencies indicated significant periods of 20.04 and 19.78 years for RAN and LAC, respectively. The phase angle for the RAN peak was found to be 41°, based on 1905, with an amplitude of 0.66C. The phase angle for the LAC peak was found to be 104°, based on 1908, with an amplitude of 0.64C; the phase angle, when adjusted by 18°/year (based on an exact 20-year period), became 50°. However, these results are not necessarily as impressive as they seem because YXD, which is approximately equidistant from RAN and LAC, showed no evidence of this period in the July temperatures.

The movements of the observing sites at LAC and RAN offered little help in identifying a reason which would have

Table 7.11. Periods (years) of significant peaks and significance levels (P,%) for monthly values at RAN.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
Jan.			18.3 3.28 2.25		none	
Feb.			3.28		18.3	21.3 2.10
Mar.		3.88	4.00			8.00 2.46
Apr.		none				11.6 2.61
May			42.7 4.00		none	
Jun.			12.8			2.37
Jul.		64.0	21.3 18.3 9.85			9.85 3.56
Aug.		none			none	
Sep.			4.92		none	
Oct.		none			2.67	5.33 2.84
Nov.	2.56	2.61	5.12		2.25	42.7
Dec.		none		2.25		8.00 2.21

explained this period, especially since the LAC site was in the same place for 65 of the 68 data years used. The missing data offered little help as well because the only temperature value missing from the RAN and LAC data sets was for November 1918 at LAC.

Table 7.12. Periods (years) of significant peaks and significance levels (P,%) for annual and seasonal values at YXD.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
Annual		none			10.7	2.25
Winter	3.28		10.7 2.21			3.20
Spring			5.12 4.00 2.61		none	
Summer	128.++		32.0	10.7	12.8	
Fall	4.92+					4.57

+ indicates that the peak exceeded the 99.5% level,
++ indicates that the peak exceeded the 99.9% level.

Very little information could be obtained from the analyses of precipitation values at the Central Alberta stations. The most noteworthy result was a 10.7-year period obtained from the analysis of summer precipitation values for YXD. This period was also the most significant peak in the YXD annual precipitation analysis. This result could hardly be considered as overwhelming. However, a brief test was performed on the summer peak because this peak accounted for 9.2% of the variance in the summer data while the same peak in the annual values accounted for 8.5% of the variance.

Data were generated for a single 10.7-year wave with a mean of 227.4mm, amplitude of 28.7mm and phase angle of 77.66° based on an origin of 1882. These values were

Table 7.13. Periods (years) of significant peaks and significance levels (P,%) for monthly values at YXD.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
Jan.		2.13	3.28		none	
Feb.			3.28		4.27 2.33	
Mar.			4.00 2.33			10.7 4.00
Apr.		3.05	2.72 2.61	3.66		6.74
May		4.00	11.6 3.88			8.53 4.13
Jun.		2.91	128. 12.8 6.40 4.74	9.85		2.98 2.13
Jul.	128++		2.03		5.33	11.6
Aug.		128.	4.27			10.7 3.88 3.12
Sep.			9.85 6.40 4.92			14.2 11.6 5.33 4.92
Oct.			6.40 2.91	2.03	2.67	
Nov.	2.56+ 4.92		4.57 2.61		32.0	
Dec.		3.12 2.46	2.21		2.25 2.21	

\+ indicates that the peak exceeded the 99.5% level,
++ indicates that the peak exceeded the 99.9% level.

Table 7.14. Periods (years) of significant peaks and significance levels (P,%) for annual and seasonal values at LAC.

	Temperature			Precipitation		
P (%)	99	97.5	95	99	97.5	95
Annual			2.67 2.61		9.85	9.14
Winter		none				4.74
Spring			3.88		none	
Summer			25.6			3.80
Fall		5.12 4.92			10.7	6.40 2.51

obtained from the analysis of summer precipitation data. The sums of the squares of the differences between the actual and generated values and between the actual values and the data mean were calculated, i.e., using a persistence forecast in the second case. The sum obtained using the generated values was 13% smaller than the sum obtained using persistence. This would seem to indicate some measure of validity in assuming the presence of a 10.7-year period.

The smoothed periodogram provided slightly different results than the raw periodogram. The raw periodogram derived 10.7- and 12.8-year peaks from the YXD summer precipitaion data. These peaks were separated by a non-significant 11.6-year period. The smoothed periodogram (using a three-point Daniell filter) derived five significant peaks among which the 11.6-year peak exceeded the 99.5% significance level. Difficulties arose in trying

Table 7.15. Periods (years) of significant peaks and significance levels (P,%) for monthly values at LAC.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
Jan.			18.3			2.84
Feb.		3.28				3.05
Mar.	3.88				3.88	
Apr.			3.05		2.61 2.56	
May		none			9.14	
Jun.		none				14.2
Jul.	64.0	21.3 18.3			3.66	
Aug.			8.00			9.85 5.12
Sep.		4.92			none	
Oct.		none				9.14
Nov.	2.56+	2.61	4.92			2.56
Dec.			3.12		none	

+ indicates that the peak exceeded the 99.5% level.

to interpret the results of the smoothed periodogram because these five peaks were no longer independent. The number of degrees of freedom used to calculate the confidence interval was approximately 4.5 for the smoothed periodogram as opposed to 1.5 for the unsmoothed periodogram. However, unlike the ACV method where a small lag value yields few spectral lines, the modified periodogram analyzes the same

number of spectral lines for both smoothed and unsmoothed results. How then are the five adjacent peaks assessed and what are the implications of smoothing? These were questions which would have proven very difficult to answer and, based on the results obtained while using the smoothed periodogram, would likely have resulted in little additional information than was available from the unsmoothed periodogram.

7.6 Southern Alberta

Tables 7.16 to 7.21 present the results obtained for LTH, YXH and MAN based on the seasonal, annual and individual monthly values of both temperature and precipitation. Only significant spectral peaks (≥95%), obtained from the unsmoothed periodogram analyses, are listed in these tables.

Neglecting the periods already discussed in Section 7.3, there remain one or two points worthy of mention concerning these last three stations.

YXH was the only station which exhibited an "unexpected" number of significant peaks on more than one occasion. Some of the occurrences involved very long periods which were readily dismissed owing to the fact that the station was moved out of the river valley in 1930. However, there remained the frequent occurrence of an 18.3- to 21.3-year period in the precipitation data, as shown in Tables 7.18 and 7.19.

Table 7.16. Periods (years) of significant peaks and significance levels (P,%) for annual and seasonal values at LTH.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
Annual		2.67				12.8 4.27
Winter		6.74 3.28				64.0
Spring		3.88			none	
Summer		12.8	42.7		12.8	
Fall		5.12 4.92	7.53		none	

A true wave was generated using a mean of 49.9mm, an amplitude of 17.3mm, a period of 19.7 years, and a phase angle of 114° based on an origin in 1884. The amplitude was obtained by taking the sum of the amplitudes of the two peaks present in the analysis of winter precipitation at YXH; this procedure was based on the assumption that the two peaks were located on either side of the true significant peak. Two sums were calculated in a manner analogous to the calculations performed on the YXD data in the previous section. The first sum, obtained by using the generated wave and the winter precipitation data, was equal to 78% of the second sum obtained using persistence. Again, the analyzed peak seemed to indicate some measure of fit when applied to the data. However, it should be noted that persistence was much better in the last 30 to 35 years of the data record.

Table 7.17. Periods (years) of significant peaks and significance levels (P,%) for monthly values at LTH.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
Jan.			6.74			64.0 7.11
Feb.		3.28	9.85			32.0 2.21
Mar.		3.88				2.78 2.25 2.21
Apr.			2.78			2.61
May			2.00		3.66	
Jun.		12.8			none	
Jul.			3.39			3.56
Aug.			12.8 8.00			2.72
Sep.			7.53 4.92		none	
Oct.		none			none	
Nov.		2.56	2.61		2.06	
Dec.			3.12 2.46		4.41	

The reason for this may possibly be seen by considering the data plot in Figure 7.8. The unsmoothed periodogram analysis of these data was shown in Figure 7.3.

The 20-year period can be seen, to a certain extent, between 1884 and 1930. Presumably, this early existence of the wave was sufficient in order to be analyzed by the

Table 7.18. Periods (years) of significant peaks and significance levels (P,%) for annual and seasonal values at YXH.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
Annual	64.0+	128.	4.92		18.3	25.6 14.2 2.37
Winter		3.28	2.46	21.3+ 18.3+	2.78	128.
Spring			128. 64.0 4.00 2.61			25.6 18.3 6.74
Summer	64.0++		128. 42.7 12.8 4.00	2.37+		12.8
Fall	4.92+		7.53			16.0

+ indicates that the peak exceeded the 99.5% level,
++ indicates that the peak exceeded the 99.9% level.

periodogram method. It is conceivable that this wave was the result of the numerous changes of location of the meteorological site in the period between 1883 and 1930. The station remained approximately 10 years at each of the first three sites and 20 years at the fourth site. The station has not been moved since 1938. Based on this information, it becomes impossible to state whether the 20-year period is truly a climatic fluctuation or results from the many relocations of the station.

Slutzky (1937), using a rigorous mathematical approach, demonstrated that series of random data could provide

Table 7.19. Periods (years) of significant peaks and significance levels (P,%) for monthly values at YXH.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
Jan.			2.13		21.3 18.3 3.05	128. 2.78
Feb.	3.28			21.3++	42.7	18.3 2.46
Mar.			4.00 2.17	18.3+	64.0	42.7 14.2
Apr.		128. 3.05			6.74	128. 3.56
May		2.17	3.56			42.7
Jun.	64.0+	42.7 12.8		2.37+		2.84
Jul.	64.0+	3.39	128. 2.03			5.57 3.88
Aug.			12.8 8.00 4.00		6.74	25.6
Sep.			4.92			8.00
Oct.	2.91		18.3			18.3 5.33
Nov.	4.92		2.56			16.0 4.92
Dec.		2.46	3.12			8.00 3.56 2.78

+ indicates that the peak exceeded the 99.5% level,
++ indicates that the peak exceeded the 99.9% level.

evidence of periodic fluctuations either throughout the whole series or in segments of the series. These findings

Table 7.20. Periods (years) of significant peaks and significance levels (P,%) for annual and seasonal values at MAN.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
Annual			21.3			64.0
Winter		3.20	16.0 7.11 3.39	64.0+		
Spring		2.46				10.7
Summer		none			none	
Fall	4.92		4.57			9.14

+ indicates that the peak exceeded the 99.5% level.

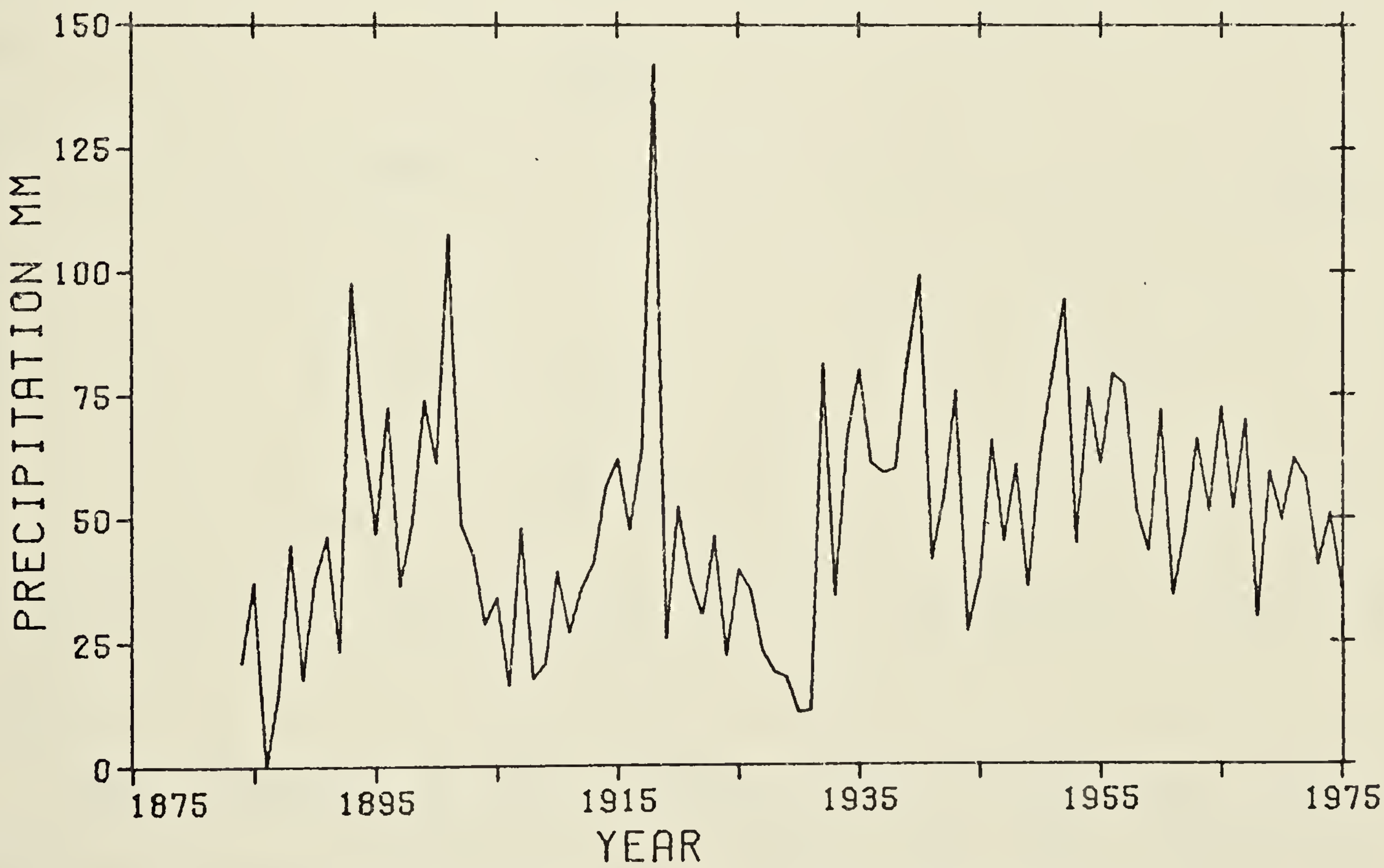

Figure 7.8. YXH winter precipitation.

were especially valid for data series which did not have totally independent elements. The analogy of this last finding to meteorological data is obvious, even if the data

Table 7.21. Periods (years) of significant peaks and significance levels (P,%) for monthly values at MAN.

	Temperature			Precipitation		
P(%)	99	97.5	95	99	97.5	95
Jan.			7.11		64.0	
Feb.	10.7	3.39			none	
Mar.		none				12.8 2.56
Apr.		none			none	
May		none		10.7	3.20	
Jun.			12.8			2.91
Jul.		64.0	3.39			3.39
Aug.			10.7 8.00			8.00
Sep.		none				3.76 3.39
Oct.		21.3			none	
Nov.		2.56	4.92		9.14	
Dec.			16.0 3.20		none	

consist of mean temperatures for a particular month or season.

The changes of the sites at LTH and MAN were not as numerous nor as drastic as the changes at YXH. Similarly, the results of the analyses for LTH and MAN were not as dramatic as those for YXH.

Missing data in the YXH record may have contributed also to the generation of erroneous results. The winter

precipitation data contained three values which had been estimated, wholly or in part, or obtained from the original published records. However, the information in these original records was rejected in compiling the microfiche data files which were described in Chapter 2. Two of the three estimated values were varied by 50mm. The resultant spectrum revealed the same significant peaks as previously mentioned. This would seem to indicate that the existence of three estimated values in this data set did not contribute significantly to the resultant spectrum. The missing values of monthly precipitation were October 1883, November-December 1885, January-April 1886, January-March 1887, August 1889, October 1889, November-December 1910 and January 1911.

No other periods were felt to be truly significant.

CHAPTER 8

SUMMARY AND CONCLUSIONS

8.1 Spectral Analysis Methods

The set of programs developed for this study provided the capability of using the modified periodogram method and the Maximum Entropy Method. The periodogram could calculate raw and smoothed values of phase, frequency and amplitude squared. Smoothing was performed by a Daniell filter. Tapering of the ends of the data was available. Three subroutines provided an interface between the spectral output and a slightly modified version of a plotting subroutine available from the Computing Services Program Library of the University of Alberta.

The unsmoothed modified periodogram method was used in all cases studied. It was found to be a highly efficient analysis technique and it performed excellently on the test data which were different combinations of cosine waves.

The smoothed phase angle and the weighted frequency proved to be quite useful in providing better estimates of the phase angle and frequency than the unsmoothed values. Tapering was found to offer no advantages. Caution should be exercised in adopting these features of the program. Analyses of test data would provide a better understanding

of its capabilities.

The smoothed modified periodogram was calculated for all data groupings. Smoothing did add a measure of stability to the results, but, for the purposes of this study, provided no improvement on the information available from the unsmoothed periodogram.

Robinson (1967) describes how the Subroutine NLOGN (see Section 4.3 and Table B.8) could be used to calculate two real-valued signals simultaneously, and to calculate the cross correlation between two real-valued signals. Both of these modifications would be desirable additions to the program prepared for this study.

The Maximum Entropy Method was used very little. This was due in part to the problems of interpreting the results, as described in Section 7.1. Also, there are the difficulties presented by inhomogeneous data. Results obtained by using MEM acquire dubious credibility given the discussion on the existence of long waves, presented in Chapter 7. This would be true even if MEM were improved beyond the technique presented by Ulrych and Clayton (1976). The greater improvement required is in the reliability of the meteorological data.

8.2 Climatological Analysis

The data used in this study were considered to be as good as any available. However, findings such as the ones presented by Schaal and Dale (1977) illustrate the degree of

inhomogeneity of the data.

Comprehensive histories of the meteorological stations are sadly lacking. The compilation of such histories for only eight stations demanded much more time than should have been necessary, but these histories were required in order to evaluate the results properly. The histories presented in Chapter 3 do not contain all the information which should be considered when making conclusions but, as indicated in Chapter 1, changes of instrumentation, observer or observing practices are frequently difficult if not impossible to ascertain.

Three observations may be made following the climatological investigations. These are the three distinct periods revealed by the analysis of seasonal temperatures.

The most notable of the three observations involves the five-year period revealed in the fall temperatures at all stations. No explanation can be formulated at this time, to dispel the existence of this period. Little evidence was found to indicate that the period was the result of leakage of power from low frequencies or aliasing of high frequencies. The occurrence of this period at all stations and the similarity of the phase angles supported the theory that this period may be quite real.

Georgiades (1977) resolved a five-year period for annual mean temperatures at Regina and Saskatoon, Saskatchewan. Only the annual mean temperatures for YXH gave evidence of a five-year period (see Table 7.1). No reasons

can be presented at this time, which would explain this apparent difference between Saskatchewan and Alberta.

The two remaining observations are not as notable as the first. However, there appears to be evidence of fairly widespread three- and four-year periods in the winter and spring mean temperatures, respectively.

Closer investigation of these three periods should be made in order to determine their possible causes and the extent of their existence across Canada.

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APPENDIX A

TOPOGRAPHICAL MAPS

This Appendix contains a set of maps, the first of which shows the geographical distribution across Alberta of the eight stations used in this study (Figure A.2); all other maps show the detailed geography at each location (Figures A.3 to A.10).

The detailed maps were constructed using the topographical series available from the Surveys and Mapping Branch, Department of Energy, Mines and Resources. All elevations are given in feet and can be converted using the scale shown in Figure A.1.

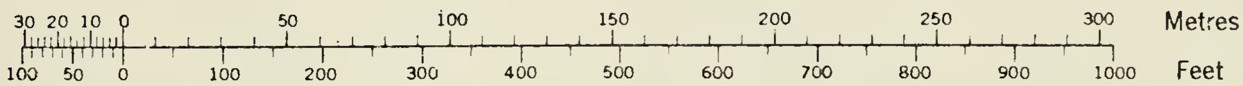
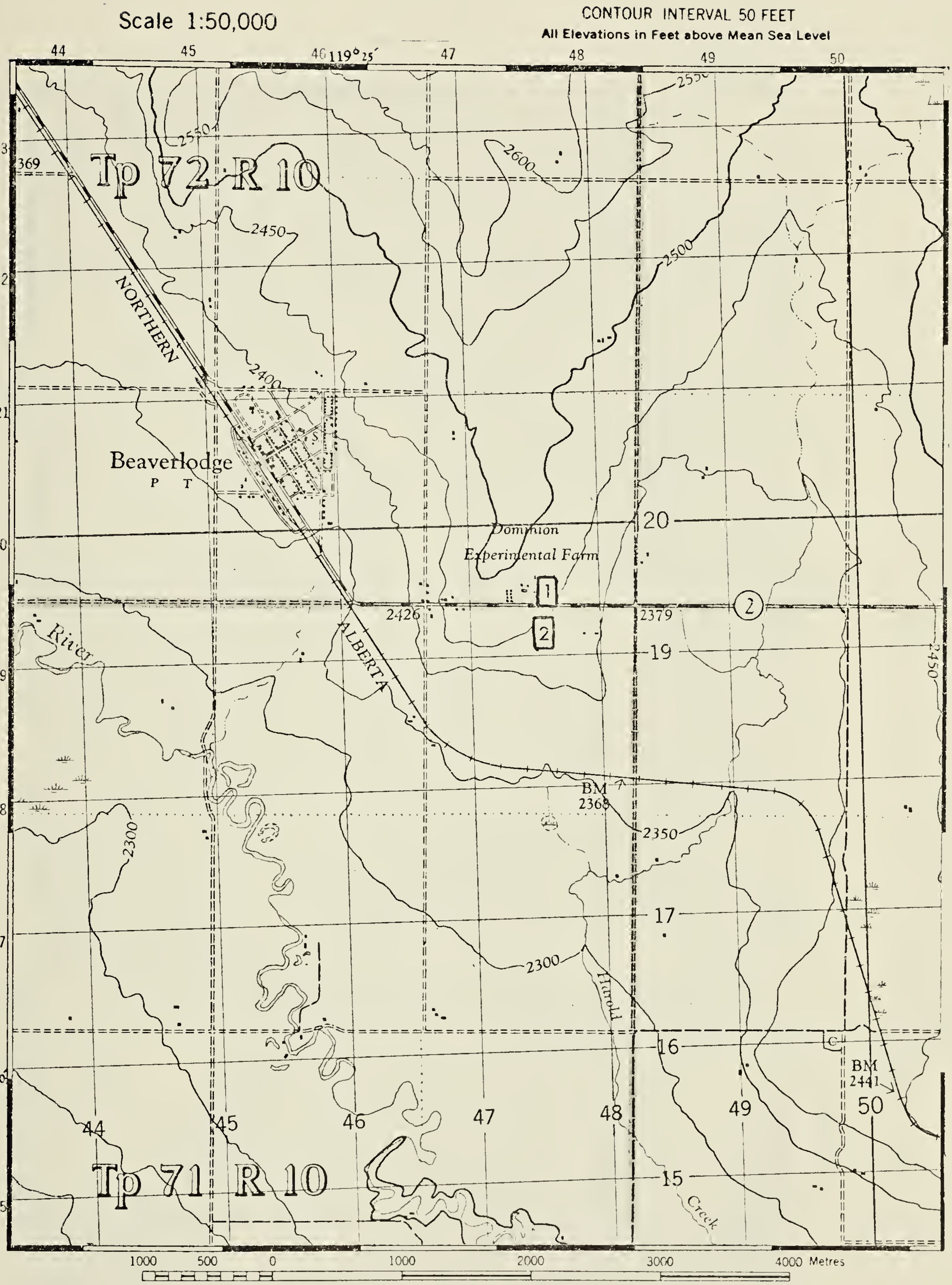


Figure A.1. Conversion scale for elevations.

Figure A.2.
Location of Alberta stations used in the study.



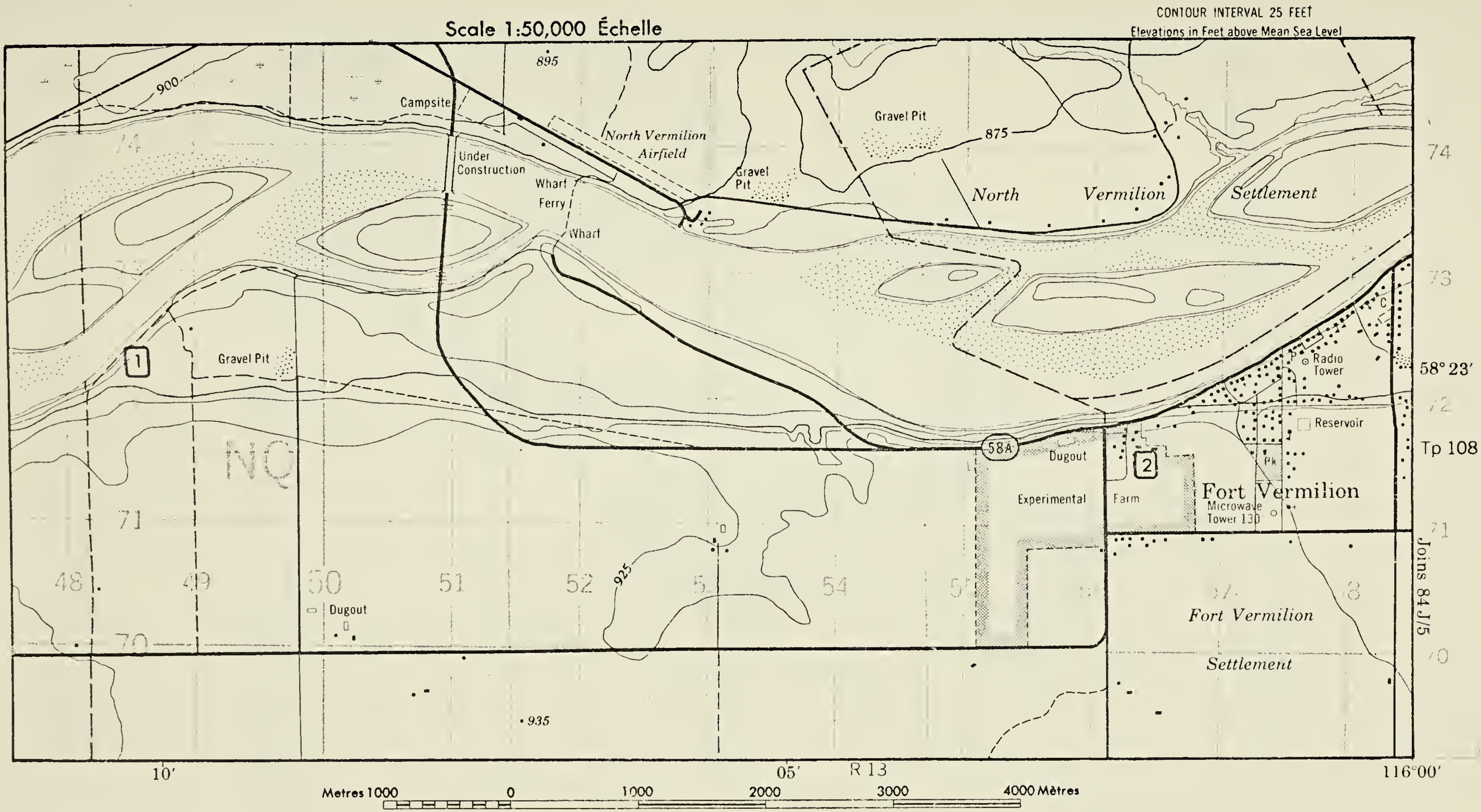
1 - (1915-1958)

2 - (1959-1975)

Figure A.3. Beaverlodge CDA site locations.

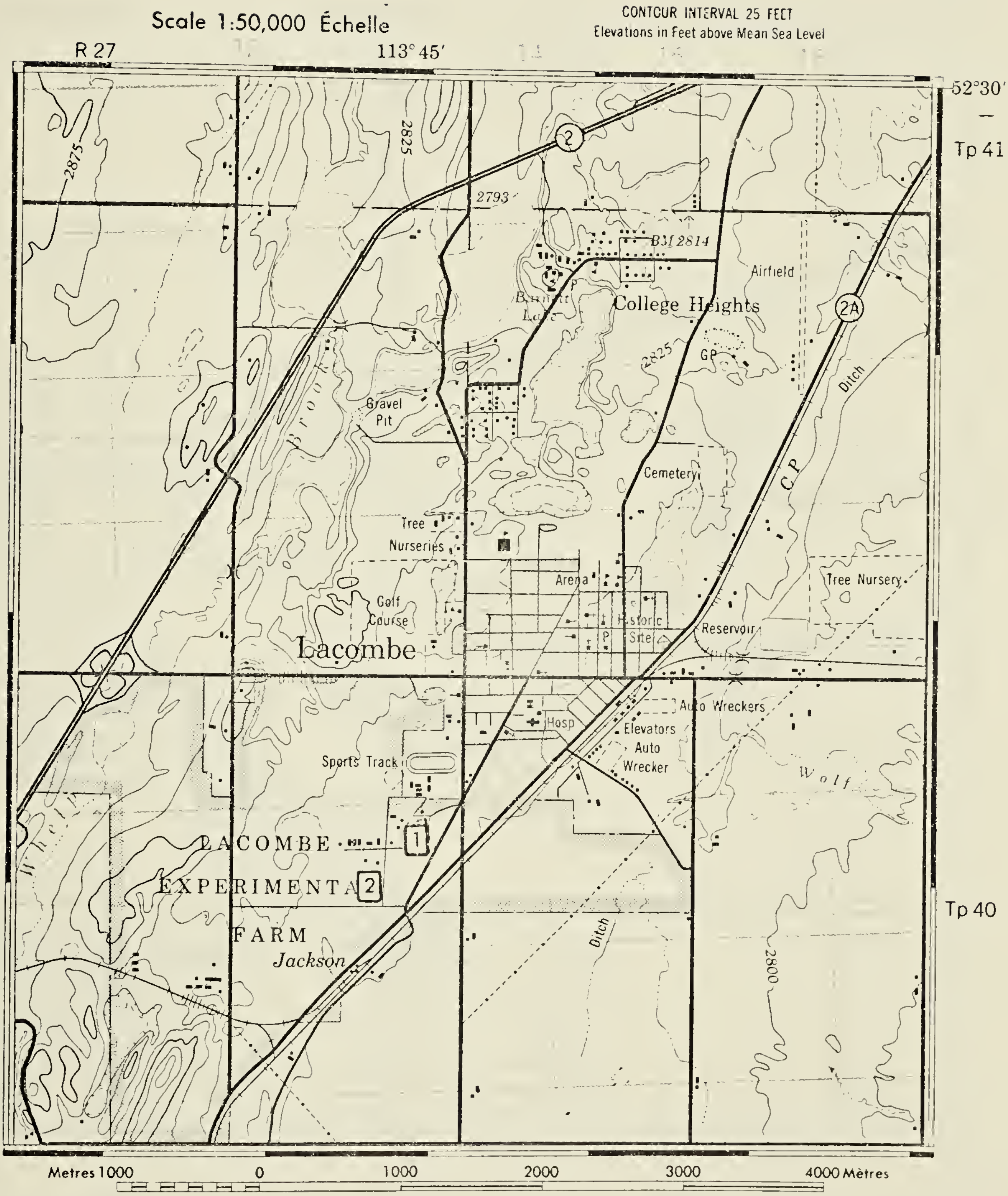
1 - (1882-1912) 2 - (1912-1942) 3 - (1937-1975)

Figure A.4. Edmonton City site locations.



1 - (1908-1935) 2 - (1936-1975)

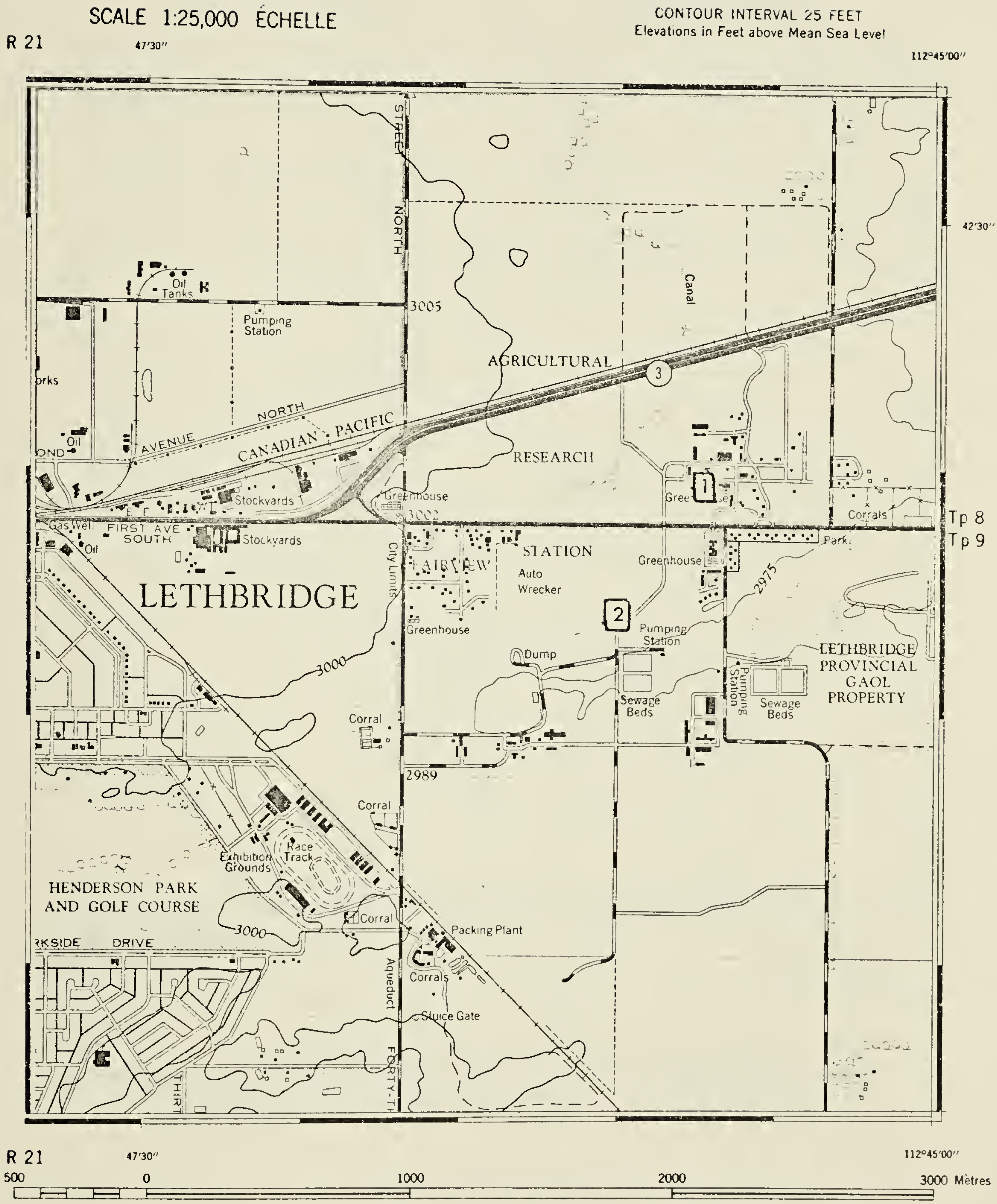
Figure A.5. Fort Vermilion CDA site locations.



1 - (1907-1972)

2 - (1972-1975)

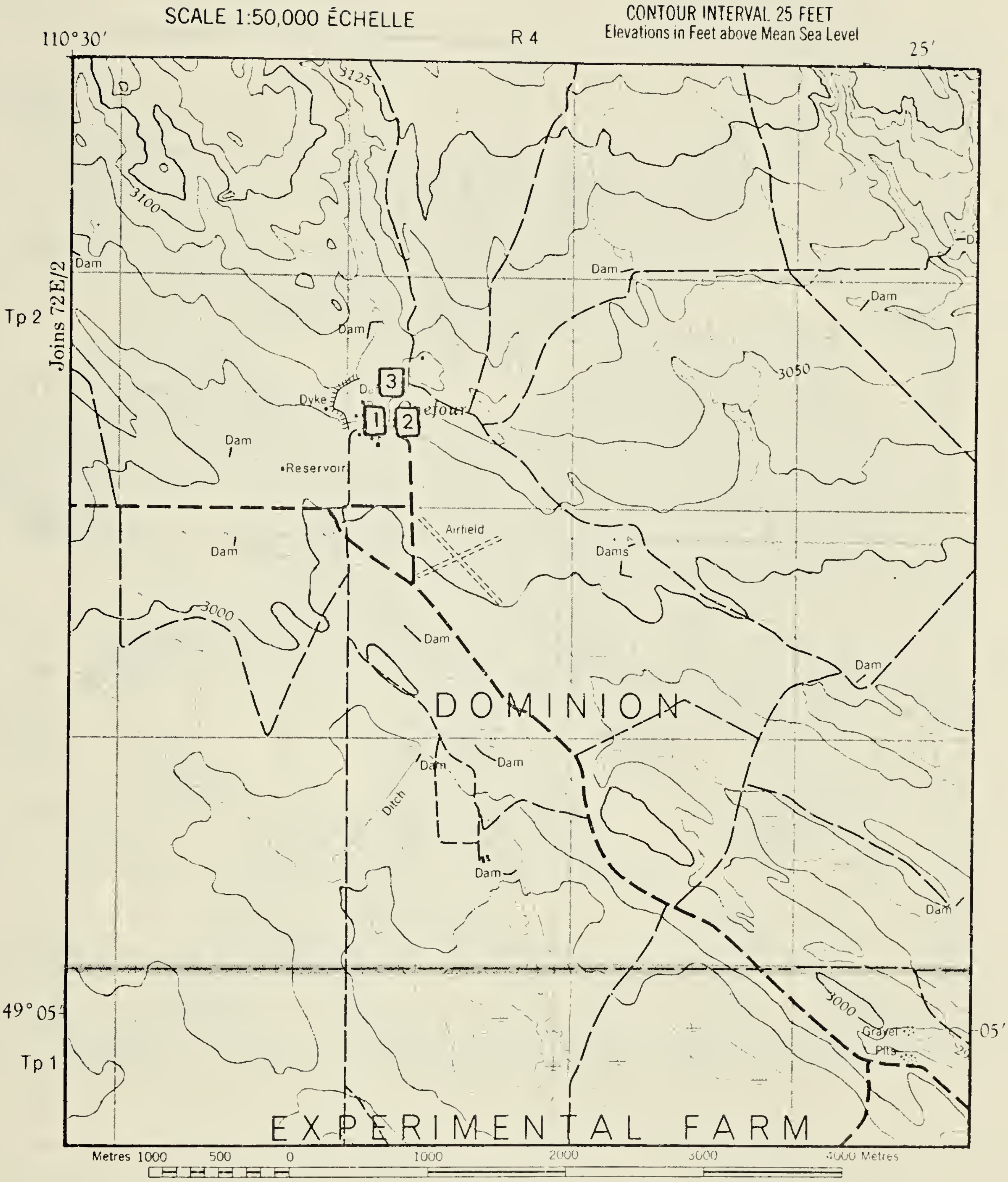
Figure A.6. Lacombe CDA site locations.



1 - (1908-1966)

2 - (1966-1975)

Figure A.7. Lethbridge CDA site locations.

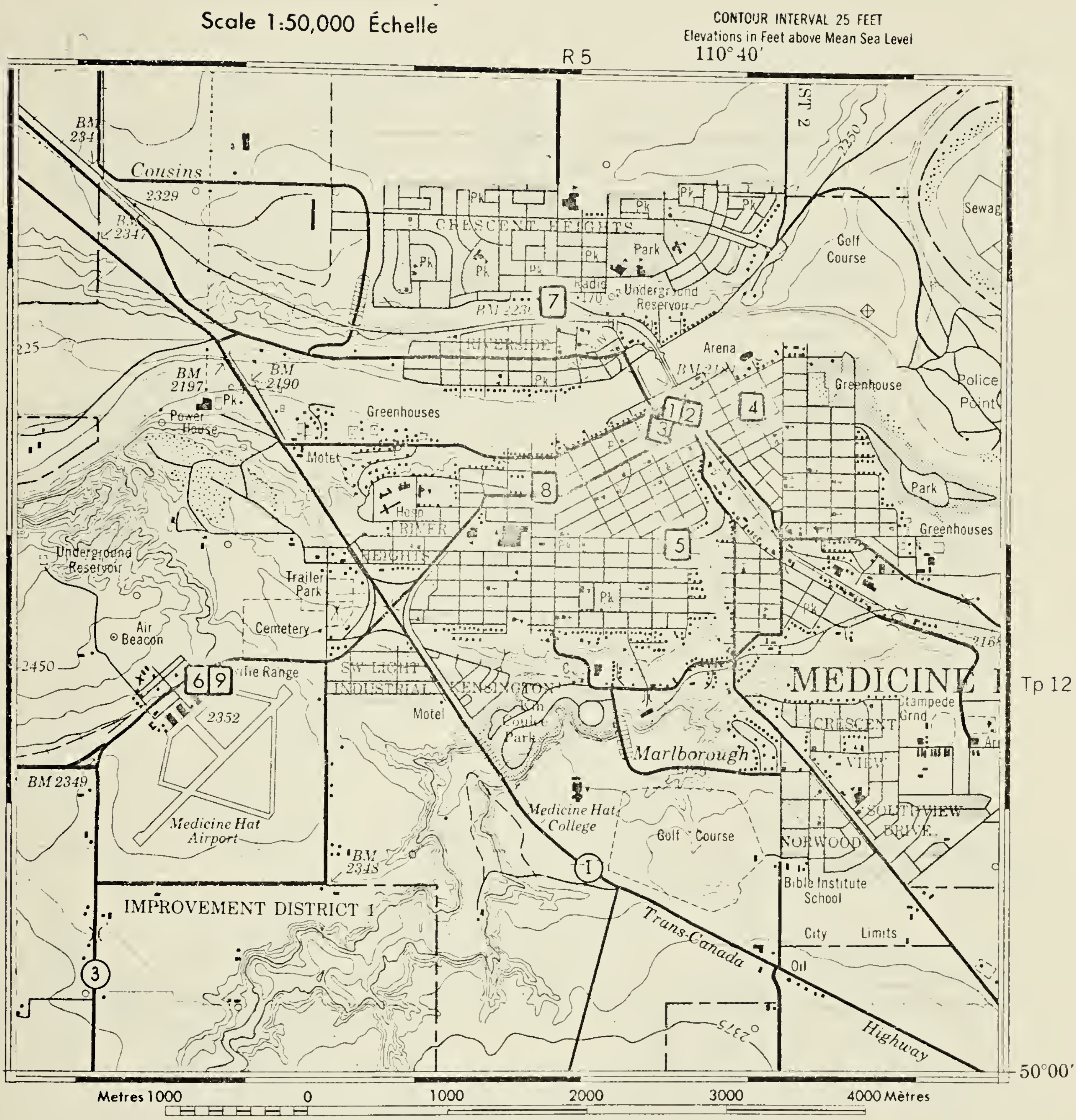


1 - (1928-1944)

2 - (1944-1949)

3 - (1949-1975)

Figure A.8. Manyberries CDA site locations.



1 - (1883-1891) 2 - (1891-1901)

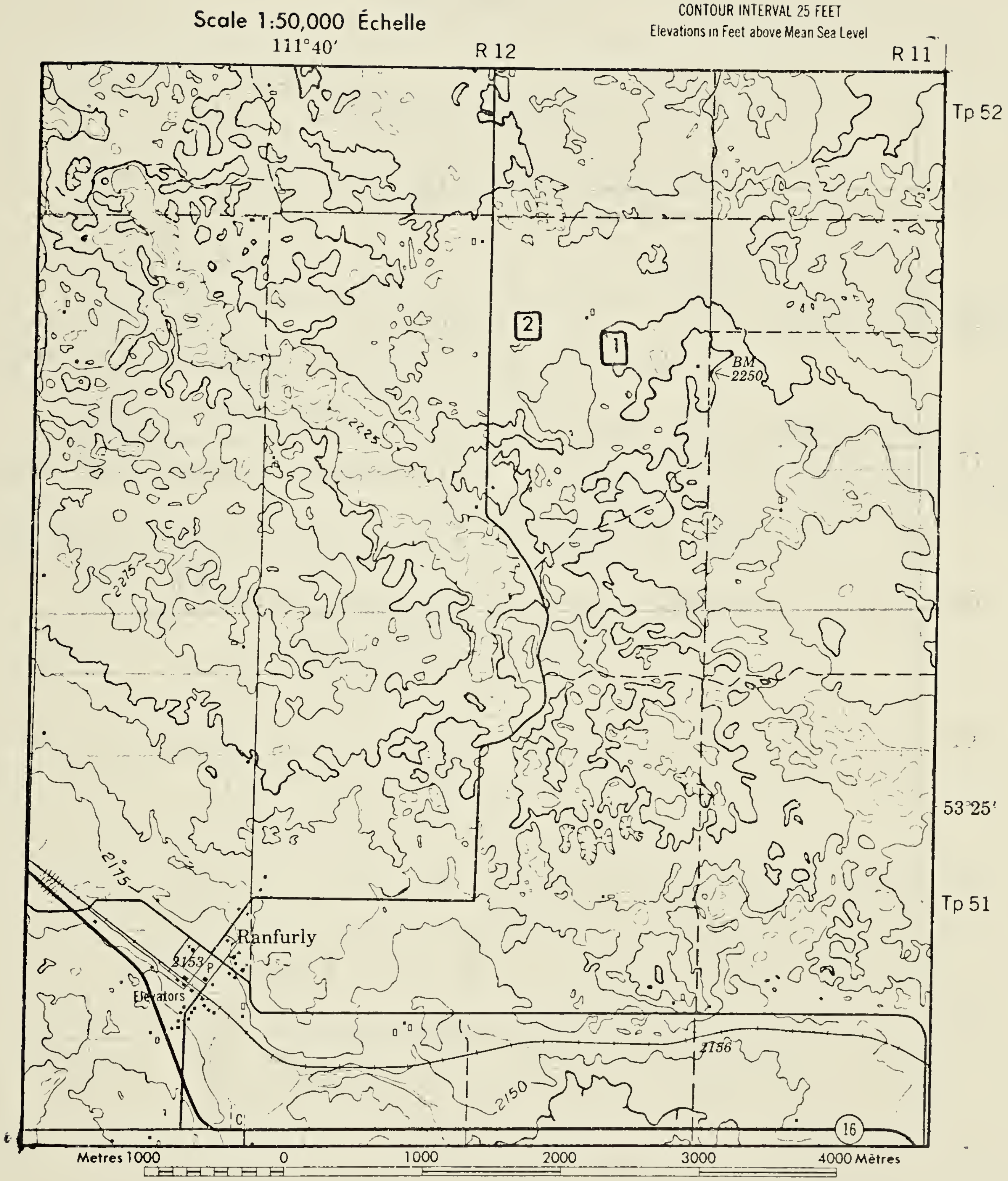
3 - (1901-1911) 4 - (1911-1930)

5 - (1930-1931) 6 - (1931-1932)

7 - (1932-1933) 8 - (1933-1938)

9 - (1938-1975)

Figure A.9. Medicine Hat site locations.



1 - (1905-1931)

2 - (1931-1975)

Figure A.10. Ranfurly site locations.

APPENDIX B

COMPUTER PROGRAMS AND SAMPLE PRINT-OUTS

This Appendix contains a copy of the computer programs used in this study (Tables B.1 to B.14).

Sample print-outs of the FFT and MEM subroutines are also included (Tables B.15 to B.19). All print-outs were obtained from the data values listed in Table B.15.

The programs were written for a FORTRAN G compiler.

Table B.1. Main program, as described in Chapter 6.

```
C       MAIN PROGRAM
C       ******************************************************************
C       FOLLOWING ARE THE PARAMETERS TO BE INPUT:
C       II = NUMBER OF DATA ON FIRST CARD(S) IF DIFFERENT FROM SUBSEQUENT
C            CARDS.
C          = 0 IF ALL CARDS ARE THE SAME.
C       NCAS = NUMBER OF DATA POINTS, MAXIMUM 4096.
C       NX = THE INTEGER POWER OF 2 USED TO GET NZERO; IT MUST BE A NUMBER
C            FROM 1 TO 12.
C       NZERO = TOTAL NUMBER OF DATA POINTS NEEDED TO HAVE AN INTEGER
C            POWER OF 2. IT MUST BE GREATER THAN OR EQUAL TO NCAS; IT
C            MUST BE ONE OF : 2,4,8,16,32,64,128,256,512,1024,2048,4096.
C       JOB = JOB NUMBER FOR IDENTIFICATION.
C       KP (POSITIVE) = NUMBER OF DATA VALUES PLOTTED AND PRINTED.
C          (NEGATIVE) = NUMBER OF DATA VALUES PRINTED ONLY.
C          = 0 NO DATA PRINTED OR PLOTTED.
C       NOTE: UNLESS ALTERED, THE PLOTTING SUBROUTINE PLTG WILL REJECT THE
C            JOB IF MORE THAN 1200 DATA POINTS ARE PASSED ON TO IT.
C       LB = NUMBER OF SPACES ADDED AT THE BEGINNING OF THE DATA
C            DISPLACING THE FIRST VALUE ON THE DATA PLOT WHICH ALLOWS THE
C            USE OF A COMMON ABSCISSA FOR PLOTS WITH DIFFERENT FIRST
C            ABSCISSA VALUES. (NOTE: NCAS+LB MUST BE LESS THAN 4097.)
C       NOM = 0 NO TAPERING OF DATA.
C           = NUMBER OF DATA POINTS TAPERED AT EACH END (INCLUDING THE
C            FILTER WEIGHTS ZERO AND ONE), MAXIMUM 100.
C       KT = +1 OR -1 FFT SPECTRAL ANALYSIS ONLY.
C          = +2 OR -2 MEM SPECTRAL ANALYSIS ONLY.
C          = +3 OR -3 FFT AND MEM.
C          (POSITIVE) SPECTRAL OUTPUT PLOTTED.
C          (NEGATIVE) NO PLOTTING OF SPECTRAL OUTPUT.
C       KN = +1 UNSMOOTHED FFT POWER PLOTTED ONLY.
C          = +2 SMOOTHED FFT POWER PLOTTED ONLY.
C          = +3 BOTH UNSMOOTHED AND SMOOTHED POWER PLOTTED.
C          (NOTE: KN IS NOT USED WHEN KT IS NEGATIVE)
C       NOTE: UNLESS ALTERED, THE PLOTTING SUBROUTINE PLTG WILL REJECT THE
C            JOB IF MORE THAN 1200 POWER VALUES ARE PASSED ON TO IT.
C       NFIL = 0 NO FILTERING OF F.F.T. RESULTS.
C            = NUMBER OF POINTS IN THE DANIELL FILTER WHICH MUST BE PLUS
C            OR MINUS 3, 5, 7, 9 OR 11.
C            (POSITIVE) FOR FILTERING ON COSINE, SINE AND FREQUENCY.
C            (NEGATIVE) FOR FILTERING ON POWER SPECTRUM AS WELL AS THE
C            OTHER THREE PARAMETERS.
C       MEMN = NUMBER OF FILTER COEFFICIENTS FOR MEM SPECTRAL ANALYSIS,
C            MAXIMUM (NCAS-2).
C            = 0 , NUMBER OF COEFFICIENTS CHOSEN AUTOMATICALLY USING
C            AKAIKE'S FINAL PREDICTION ERROR.
C       MEMX = MULTIPLICATION FACTOR FOR EXPANDING THE LOWER END OF THE
C            FREQUENCY SPECTRUM. SET EQUAL TO ONE IF NO EXPANSION DESIRED.
C       SIGN = +1.0 OR -1.0 DETERMINES THE SIGN OF THE EXPONENTIAL
C            FUNCTION OF THE TRANSFORM OF THE FFT.
C       DT = TIME INTERVAL OF THE DATA IN UNITS CHOSEN FOR THE PROBLEM.
C       FOR1 AND FOR2 ARE VARIABLE FORMATS (MAX. 60 CHARACTERS):
C            FOR1 READS THE FIRST II VALUES (OMIT IF II = 0),
C            FOR2 READS ALL OTHER VALUES.
C       ******************************************************************
```

```
  100 FORMAT(13I5,F5.1,F5.3)
  101 FORMAT(1H1,'***** NX AND NZERO ARE NOT COMPATIBLE *****')
  102 FORMAT(15A4)
  103 FORMAT(1H1,'JOB NUMBER',I6,' WITH',I5,' VALUES USED OF WHICH',I5,'
     * VALUES ARE PLOTTED AND/OR PRINTED; THE SIGN FOR THE FFT IS',F5.1,
     *///)
  104 FORMAT(1H0,'RAW DATA MEAN = ',F10.3,' RAW DATA VARIANCE = ',F13.3,
     *' RAW DATA STANDARD DEVIATION = ',F10.3,/)
  105 FORMAT(1H0,'POINT NO. ',12I8)
  106 FORMAT(1H ,'VALUE',6X,12F8.2)
  107 FORMAT(1H1,'***** YOU BLEW IT, KT = 0 *****')
C     *******************************************************************
C
      DIMENSION DATA(4096),FOR1(15),FOR2(15),V(4096),W(4096),PLOT(4096)
    5 READ(5,100,END=95)II,NCAS,NX,NZERO,JOB,KP,LB,NOM,KT,KN,NFIL,MEMN,
     *MEMX,SIGN,DT
      NZO=2**NX
      IF(NZO.EQ.NZERO)GO TO 10
      WRITE(6,101)
      STOP
   10 IF(II.EQ.0)GO TO 15
      READ(5,102)FOR1
   15 READ(5,102)FOR2
      IF(II.EQ.0)GO TO 20
      READ(5,FOR1)(DATA(L),L=1,II)
   20 IM=II+1
      READ(5,FOR2)(DATA(L),L=IM,NCAS)
C
C     FIND THE VARIANCE, MEAN AND STANDARD DEVIATION OF THE DATA.
C
      CALL VARSD(NCAS,DATA,EM,S,SD)
C
C     PRINT OUT THE JOB PARTICULARS.
C
      KQ=IABS(KP)
      WRITE(6,103)JOB,NCAS,KQ,SIGN
      WRITE(6,104)EM,S,SD
C
C     PRINT OUT DATA IF REQUIRED.
C
      IF(KP.EQ.0)GO TO 40
      MN=1
   25 NN=MN+11
      IF(NN.GT.KQ)NN=KQ
      WRITE(6,105)(J,J=MN,NN)
      WRITE(6,106)(DATA(I),I=MN,NN)
      MN=MN+12
      IF(MN.LE.KQ)GO TO 25
      IF(KP.LT.0)GO TO 40
C
C     PLOT THE DATA.
C     THE FIRST (LB) DATA VALUES ARE SET TO VALUES WHICH ARE HOPEFULLY
C          BEYOND THE RANGE OF THE PLOT.
C
      IF(LB.LT.0)LB=0
      DO 30 I=1,NCAS
   30 PLOT(I+LB)=DATA(I)
      IF(LB.LE.0)GO TO 38
      DO 35 I=1,LB
   35 PLOT(I)=1000000.0
```

```
   38 NTOT=NCAS+LB
      CALL PLTDAT(PLOT,NTOT)
C
C     REMOVE THE MEAN FROM THE DATA FOR USE BY THE F.F.T. AND STORE
C          IN ARRAY V.
C     NORMALIZE THE DATA ARRAY FOR THE M.E.M. AND STORE IN ARRAY W.
C
   40 DO 45 I=1,NCAS
      V(I)=DATA(I)-EM
   45 W(I)=V(I)/SD
C
C     NOW PERFORM THE ANALYSIS REQUIRED AND THE PLOTTING.
C     IF BOTH FFT AND MEM OUTPUTS ARE PLOTTED, SPECIFY PARAMETERS FOR
C          THE FFT PLOT FIRST. IF BOTH UNSMOOTHED AND SMOOTHED POWER
C          SPECTRUM FROM THE FFT ARE PLOTTED, SPECIFY PARAMETERS FOP THE
C          UNSMOOTHED SPECTRUM FIRST.
C
      KT2=KT+4
      NZP=NZERO/2+1
      GO TO(50,55,60,65,70,75,80),KT2
   50 CALL FASTFT(NCAS,NX,NZERO,NOM,NFIL,SIGN,DT,V)
      CALL BURGME(NCAS,DT,W,MP,MEMN,MEMX)
      GO TO 90
   55 CALL BURGME(NCAS,DT,W,MP,MEMN,MEMX)
      GO TO 90
   60 CALL FASTFT(NCAS,NX,NZERO,NOM,NFIL,SIGN,DT,V)
      GO TO 90
   65 WRITE(6,107)
      GO TO 90
   70 CALL FASTFT(NCAS,NX,NZERO,NOM,NFIL,SIGN,DT,V)
      CALL PLTFFT(NZP,KN)
      GO TO 90
   75 CALL BURGME(NCAS,DT,W,MP,MEMN,MEMX)
      CALL PLTMEM(MP)
      GO TO 90
   80 CALL FASTFT(NCAS,NX,NZERO,NOM,NFIL,SIGN,DT,V)
      CALL PLTFFT(NZP,KN)
      CALL BURGME(NCAS,DT,W,MP,MEMN,MEMX)
      CALL PLTMEM(MP)
   90 CONTINUE
      GO TO 5
   95 STOP
      END
```

Table B.2. Subroutine VARSD, as described in Chapter 6.

```
      SUBROUTINE VARSD(NCAS,DATA,EM,S,SD)
C     *****************************************************************
C     FIND THE VARIANCE AND STANDARD DEVIATION.
C     *****************************************************************
  110 FORMAT(1H1,20X,'***** ZERO OR NEGATIVE VARIANCE *****')
C     *****************************************************************
C
      DIMENSION DATA(NCAS)
      RCA=FLOAT(NCAS)
      SSQ=0.0
      EMN=0.0
      DO 10 I=1,NCAS
      EMN=EMN+DATA(I)
   10 SSQ=SSQ+DATA(I)*DATA(I)
      EM=EMN/RCA
      S=SSQ/RCA-EM*EM
      IF(S.GT.0.)GO TO 20
      S=0.0
      WRITE(6,110)
      SD=0.0
      RETURN
   20 SD=SQRT(S)
      RETURN
      END
```

Table B.3. Subroutine FASTFT, as described in Chapter 4.

```
      SUBROUTINE FASTFT(NCAS,NX,NZERO,NCM,NFIL,SIGN,DT,DATA)
C     ************************************************************************
C     X IS AN ARRAY OF COMPLEX NUMBERS.
C     A LABELLED COMMON BLOCK WILL HOLD THE RESULTS FOR PLOTTING.
C
C     ADD ZEROS AT THE BEGINNING AND/OR THE END OF THE DATA TO OBTAIN
C          THE REQUIRED LENGTH NZERO.
C     FIND THE NEW MEAN AND STANDARD DEVIATION, AND NORMALIZE THE DATA.
C     ************************************************************************
  119 FORMAT(1H1,'***** NZERO TOO SMALL *****')
  120 FORMAT(1H1,'FOR THE F.F.T., THE MEAN = ',F10.6,' TOTAL VARIANCE =
     *',F13.5,' STANDARD DEVIATION = ',F10.4,/,'  AFTER APPLICATION OF A
     *',I4,' POINT TAPER.')
  121 FORMAT(1H-,12X,'TIME',8X,'REAL X',5X,'IMAGINARY X',/)
  122 FORMAT(1H ,11X,I4,2(7X,F8.3))
  123 FORMAT(1H-,' ALL SMOOTHING APPLIED USING A',I3,' POINT DANIELL (BO
     *XCAR) FILTER',//,58X,'UNSMOOTHED      SMOOTHED      SMOOTHED      W
     *EIGHTED      SMOOTHED',/,'  FREQUENCY      COSINE',10X,'SINE',10X,'
     *PHASE',8X,'POWER',10X,'POWER',8X,'AMPL. SQ.    FREQUENCY      PHASE
     *',//)
  124 FORMAT(1H ,F9.5,1X,2E15.5,F12.3,3E15.5,F11.5,F12.3)
  125 FORMAT(1H0,' THE SUMS ARE',39X,3E15.5,//,'  ALSO NOTE THAT THE FIR
     *ST AND LAST',I3,' POINT(S) OF THE SMOOTHED POWER, AMPL. SQ., PHASE
     *',/,'  AND WEIGHTED FREQUENCY COLUMNS HAVE NOT BEEN SMOOTHED')
  126 FORMAT(1H-,' ALL SMOOTHING APPLIED USING A',I3,' POINT DANIELL (BO
     *XCAR) FILTER',//,58X,'UNSMOOTHED    UNSMOOTHED    WEIGHTED     SMO
     *OTHED',/,'  FREQUENCY       COSINE',10X,'SINE',10X,'PHASE',8X,'POWE
     *R',8X,'AMPL. SQ.     FREQUENCY      PHASE',//)
  127 FORMAT(1H ,F9.5,1X,2E15.5,F12.3,2E15.5,F11.5,F12.3)
  128 FORMAT(1H0,' THE SUMS ARE',39X,2E15.5,//,'  ALSO NOTE THAT THE FIR
     *ST AND LAST',I3,' POINT(S) OF THE SMOOTHED PHASE',/,'  AND WEIGHTE
     *D FREQUENCY COLUMNS HAVE NOT BEEN SMOOTHED')
  129 FORMAT(1H-,' NO SMOOTHING HAS BEEN APPLIED',//,10X,'FREQUENCY',7X,
     *'COSINE',12X,'SINE',12X,'PHASE',10X,'POWER',10X,'AMPL. SQ.',//)
  130 FORMAT(1H ,8X,F9.5,2E17.5,F14.3,2E17.5)
  131 FORMAT(1H0,' THE SUMS ARE',52X,2E17.5)
C     ************************************************************************
C
      DIMENSION CX(2050),SX(2050),PHZ(2050),DATA(NCAS),XX(4096),XC(2050)
     *,XS(2050),WOP(2050),ZHP(2050),QERF(2050),AMPSQ(2050)
      COMPLEX X(4096)
      COMMON /OUTPLT/ FREQ(2050),POW(2050),SOW(2050)
C
C     TAPER THE DATA AT BOTH ENDS (IF DESIRED) USING A COSINE BELL.
C
      IF(NOM.LE.0)GO TO 16
      CALL TAPER(NCAS,DATA,NCM)
C
C     CALCULATE THE MEAN AND VARIANCE AND ASSIGN THE DATA TO THE COMPLEX
C          ARRAY X.
C
   16 DO 17 I=1,NCAS
   17 XX(I)=DATA(I)
      IF(NZERO.EQ.NCAS)GO TO 30
      IF(NZERO.GT.NCAS)GO TO 20
```

```
      WRITE(6,119)
      RETURN
   20 MNM=NCAS+1
      DO 25 I=MNM,NZERO
   25 XX(I)=0.0
   30 CALL VARSD(NZERO,XX,EMM,SS,SD)
      WRITE(6,120) EMM,SS,SD,NOM
      DO 35 I=1,NZERO
   35 X(I)=(XX(I)-EMM)/SD
      CALL NLOGN(NX,NZERO,X,SIGN)
C
C     WE NOW HAVE THE FOURIER TRANSFORM OF THE DATA STORED IN ARRAY X.
C
      IF(SIGN.LT.0.0)GO TO 50
      WRITE(6,121)
      DO 45 I=1,NZERO
   45 WRITE(6,122) I,X(I)
      RETURN
   50 RNZ=FLOAT(NZERO)*DT
      NZP=NZERO/2+1
      DO 60 I=1,NZP
      CX(I)=REAL(X(I))
      SX(I)=AIMAG(X(I))
   60 FREQ(I)=FLOAT(I-1)/RNZ
C
C     CALCULATE THE POWER AND PHASE.
C
      CALL POLAR(NZP,CX,SX,POW,PHZ)
      FAM=2.0*SS*FLOAT(NZERO)/FLOAT(NCAS)
C
C     APPLY DANIELL FILTER WHERE REQUIRED.
C     A MINIMUM VALUE OF .000000011 IS ASSIGNED TO THE POWER (AFTER
C          PRINTING) IN ORDER TO PREVENT ERRORS IN THE PLOTTING
C          PROCEDURE.
C
      BX=.11E-08
      PSUM=0.0
      SSUM=0.0
      ASUM=0.0
      IF(NFIL.EQ.0)GO TO 80
      MFIL=IABS(NFIL)
      KA=(MFIL-1)/2
      CALL DANI(MFIL,NZP,CX,XC)
      CALL DANI(MFIL,NZP,SX,XS)
      CALL POLAR(NZP,XC,XS,WCP,ZHP)
      CALL DANY(MFIL,NZP,FREQ,POW,QERF)
      IF(NFIL.GT.0)GO TO 70
      CALL DANI(MFIL,NZP,POW,SOW)
C
C     THE RESULTS ARE PRINTED.
C
      WRITE(6,123) MFIL
      DO 66 I=1,NZP
      AMPSQ(I)=FAM*SOW(I)
      WRITE(6,124) FREQ(I),CX(I),SX(I),PHZ(I),POW(I),SOW(I),AMPSQ(I),QERF
     *(I),ZHP(I)
      PSUM=PSUM+POW(I)
      SSUM=SSUM+SOW(I)
      ASUM=ASUM+AMPSQ(I)
      IF(POW(I).LT.BX) POW(I)=BX
```

```
      IF(SOW(I).LT.BX)SOW(I)=BX
   66 CONTINUE
      WRITE(6,125)PSUM,SSUM,ASUM,KA
      RETURN
   70 WRITE(6,126)MFIL
      DO 75 I=1,NZP
      AMPSQ(I)=FAM*POW(I)
      WRITE(6,127)FREQ(I),CX(I),SX(I),PHZ(I),POW(I),AMPSQ(I),QERF(I),ZHP
     *(I)
      PSUM=PSUM+POW(I)
      ASUM=ASUM+AMPSQ(I)
      IF(POW(I).LT.BX)POW(I)=BX
   75 CONTINUE
      WRITE(6,128)PSUM,ASUM,KA
      RETURN
   80 WRITE(6,129)
      DO 85 I=1,NZP
      AMPSQ(I)=FAM*POW(I)
      WRITE(6,130)FREQ(I),CX(I),SX(I),PHZ(I),POW(I),AMPSQ(I)
      PSUM=PSUM+POW(I)
      ASUM=ASUM+AMPSQ(I)
      IF(POW(I).LT.BX)POW(I)=BX
   85 CONTINUE
      WRITE(6,131)PSUM,ASUM
      RETURN
      END
```

Table B.4. Subroutine DANI, as described in Chapter 4.

```
      SUBROUTINE DANI(NFIL,N,POW,BMP)
C     ******************************************************************
C     NFIL = NUMBER OF FILTER POINTS: 3, 5, 7, 9 OR 11.
C     N = NZERO/2 + 1.
C     POW = UNFILTERED ARRAY INPUT.
C     BMP = FILTERED ARRAY OUTPUT.
C
C     NOTE THAT THE FIRST AND LAST (NFIL-1)/2 POINTS OF THE ARRAY WILL
C          EMERGE UNFILTERED.
C     ******************************************************************
  400 FORMAT(1H1,'***** NUMBER OF FILTER POINTS EXCEEDS 11 *****')
C     ******************************************************************
C
      DIMENSION POW(N),BMP(N)
      IF(NFIL.GT.11)GO TO 50
      DO 10 M=1,N
   10 BMP(M)=0.0
      FIL=FLOAT(NFIL)
      KA=(NFIL-1)/2
      NA=KA+1
      NB=N-KA
      DO 30 I=NA,NB
      NC=I-KA
      ND=I+KA
      DO 20 J=NC,ND
   20 BMP(I)=BMP(I)+POW(J)
   30 BMP(I)=BMP(I)/FIL
      NK=N+1
      DO 40 I=1,KA
      BMP(I)=POW(I)
   40 BMP(NK-I)=POW(NK-I)
      RETURN
   50 WRITE(6,400)
      RETURN
      END
```

Table B.5. Subroutine DANY, as described in Chapter 4.

```
      SUBROUTINE DANY(NFIL,N,FR,PO,QE)
C     ******************************************************************
C     MODIFIED DANIELL FILTER FOR RESOLVING THE PEAK FREQUENCIES USING
C          THE POWER AS A WEIGHTING FACTOR.
C
C     NFIL = NUMBER OF FILTER POINTS: 3, 5, 7, 9 OR 11.
C     N = NZERO/2 + 1.
C     FR = ARRAY TO BE FILTERED.
C     PO = ARRAY USED AS WEIGHTS.
C     QE = FILTERED ARRAY OUTPUT.
C
C     NOTE THAT THE FIRST AND LAST (NFIL-1)/2 POINTS OF THE ARRAY WILL
C          EMERGE UNFILTERED.
C     ******************************************************************
  450 FORMAT(1H1,'***** NUMBER OF FILTER POINTS EXCEEDS 11 *****')
C     ******************************************************************
C
      DIMENSION FR(N),PO(N),QE(N)
      IF(NFIL.GT.11)GO TO 50
      DO 10 M=1,N
   10 QE(M)=0.0
      FIL=FLOAT(NFIL)
      KA=(NFIL-1)/2
      NA=KA+1
      NB=N-KA
      DO 30 I=NA,NB
      NC=I-KA
      ND=I+KA
      POW=0.0
      DO 20 J=NC,ND
      QE(I)=QE(I)+FR(J)*PO(J)
   20 POW=POW+PO(J)
   30 QE(I)=QE(I)/POW
      NK=N+1
      DO 40 I=1,KA
      QE(I)=FR(I)
   40 QE(NK-I)=FR(NK-I)
      RETURN
   50 WRITE(6,450)
      RETURN
      END
```

Table B.6. Subroutine TAPER, as described in Chapter 4.

```
      SUBROUTINE TAPER(N,X,LC)
C     ***********************************************************************
C     N = TOTAL SIZE OF ARRAY X.
C     X = DATA ARRAY.
C     LC = NUMBER OF FILTER POINTS IN HALF THE FILTER, MAXIMUM 100.
C     ***********************************************************************
C
      DIMENSION X(N),T(100)
      PI=3.141592654
      CM=FLOAT(LC-1)
      DO 10 I=1,LC
   10 T(I)=(COS(PI*FLOAT(I-LC)/CM)+1.0)/2.0
C
C          T(1) = 0.0 , T(LC) = 1.0
C
      DO 20 I=1,LC
      K=N-I+1
      X(I)=X(I)*T(I)
   20 X(K)=X(K)*T(I)
      RETURN
      END
```

Table B.7. Subroutine POLAR, as described in Chapter 4.

```
      SUBROUTINE POLAR(L,RE,XIM,AMP,PHZ)
C     ******************************************************************
C     OBTAINS THE PHASE AND THE POWER (AMPLITUDE SQUARED).
C     ******************************************************************
C
      DIMENSION RE(L),XIM(L),AMP(L),PHZ(L)
      PI=3.141592654
      ANG=180./PI
      DO 120 I=1,L
      AMP(I)=2.0*(RE(I)*RE(I)+XIM(I)*XIM(I))
      IF(XIM(I)) 10,20,30
   10 IF(RE(I)) 40,50,60
   20 IF(RE(I)) 70,80,60
   30 IF(RE(I)) 100,90,60
   40 PHZ(I)=ATAN(XIM(I)/RE(I))-PI
      GO TO 110
   50 PHZ(I)=-90.0
      GO TO 120
   60 PHZ(I)=ATAN(XIM(I)/RE(I))
      GO TO 110
   70 PHZ(I)=-180.0
      GO TO 120
   80 PHZ(I)=0.0
      GO TO 120
   90 PHZ(I)=90.0
      GO TO 120
  100 PHZ(I)=ATAN(XIM(I)/RE(I))+PI
  110 PHZ(I)=PHZ(I)*ANG
  120 CONTINUE
      RETURN
      END
```

Table B.8. Subroutine NLOGN, as described in Chapter 4.

```
      SUBROUTINE NLOGN(N,LX,X,SIGN)
C     ******************************************************************
C     THE RETURN OUTPUT IN ARRAY X IS THE FOURIER TRANSFORM OF THE DATA.
C     M IS DIMENSIONED TO HAVE AS MANY VALUES AS THE LARGEST VALUE OF N.
C     (FROM ROBINSON, 1967)
C     ******************************************************************
C
      DIMENSION M(12)
      COMPLEX X(LX),WK,HOLD,Q,CMPLX
      DO 10 I=1,N
   10 M(I)=2**(N-I)
      FLX=FLOAT(LX)
      PIX=SIGN*2.*3.141592654/FLX
      DO 40 L=1,N
      NBL=2**(L-1)
      LBL=LX/NBL
      LBH=LBL/2
      K=0
      DO 40 IBL=1,NBL
      FK=K
      V=PIX*FK
      WK=CMPLX(COS(V),SIN(V))
      IST=LBL*(IBL-1)
      DO 20 I=1,LBH
      J=IST+I
      JH=J+LBH
      Q=X(JH)*WK
      X(JH)=X(J)-Q
   20 X(J)=X(J)+Q
      DO 30 I=2,N
      II=I
      IF(K.LT.M(I))GO TO 40
   30 K=K-M(I)
   40 K=K+M(II)
      K=0
      DO 70 J=1,LX
      IF(K.LT.J)GO TO 50
      HOLD=X(J)
      X(J)=X(K+1)
      X(K+1)=HOLD
   50 DO 60 I=1,N
      II=I
      IF(K.LT.M(I))GO TO 70
   60 K=K-M(I)
   70 K=K+M(II)
      IF(SIGN.GT.0.0)RETURN
      DO 80 I=1,LX
   80 X(I)=X(I)/FLX
      RETURN
      END
```

Table B.9. Subroutine BURGME, as described in Chapter 5.

```
      SUBROUTINE BURGME(N,DT,DATA,NZP,MEMN,MEMX)
C     *******************************************************************
C     USING THE MAXIMUM ENTROPY METHOD, WE NOW CALCULATE A DIFFERENT SET
C          OF SPECTRAL ESTIMATES.
C     THE AKAIKE FINAL PREDICTION ERROR IS USED TO DETERMINE THE OPTIMUM
C          NUMBER OF FILTER COEFFICIENTS. IF THE NUMBER OF COEFFICIENTS
C          IS SPECIFIED, THE AKAIKE FPE IS BYPASSED, AND FPE(M) AND
C          RFPE(M) ARE SET AS ZERO.
C
C     N = NUMBER OF DATA POINTS.
C     DT = TIME INTERVAL CHOSEN AS BEFORE.
C     DATA = NORMALIZED DATA.
C     NZP = NUMBER OF FREQUENCY AND POWER VALUES CALCULATED.
C     MEMN = NUMBER OF FILTER COEFFICIENTS IF NOT CHOSEN AUTOMATICALLY
C          BY AKAIKE'S FPE, MAXIMUM (N-2).
C     MEMX = EXPANSION FACTOR FOR LOWER END OF FREQUENCY SPECTRUM.
C
C     A LABELLED COMMON BLOCK WILL HOLD THE RESULTS FOR PLOTTING.
C     *******************************************************************
  145 FORMAT(1H1,'THE NUMBER OF FILTER COEFFICIENTS CALCULATED IS',I5,'.
     * THE EXPANSION FACTOR IS',I5,'. DT = ',F5.3)
  146 FORMAT(1H1,'THE NUMBER OF FILTER COEFFICIENTS WILL BE CHOSEN AUTOM
     *ATICALLY. THE EXPANSION FACTOR IS',I5,'. DT = ',F5.3)
  147 FORMAT(1H-,'THE PROCESS OF CONVERGENCE TO MINIMUM FPE NOW FOLLOWS'
     *)
  148 FORMAT(1H0,4X,'M          FPE(M)            P(M)',/)
  149 FORMAT(1H ,I5,2E15.5)
  150 FORMAT(1H1,'FOR M = ',I5,' WE HAVE FPE(M) = ',E15.5,' AND P(M) = '
     *,E15.5,/,'THE FILTER COEFFICIENTS ARE ')
  151 FORMAT(1H ,6F10.5)
  152 FORMAT(1H1,'FPE(0) = ',E15.5,' FPE(M) = ',E15.5,' RELATIVE VALUE (
     *RFPE(M)) = ',E15.5,/)
  153 FORMAT(1H0,12X,'FREQUENCY',11X,'POWER',/)
  154 FORMAT(1H ,F20.5,E20.5)
C     *******************************************************************
C
      DIMENSION DATA(N),YA(4096),YB(4096),AA(4096),AB(4096),CA(4096)
      COMMON /MEMOUT/ FREQ(4097),P(4097)
C
C     INITIALIZE THE PROCESS.
C
      MT=25
      RCA=FLOAT(N)
      IS=0
      SUM=0.0
      DO 15 I=1,N
   15 SUM=SUM+DATA(I)*DATA(I)
      PM=SUM/RCA
      FPEO=PM*FLOAT(N+1)/FLOAT(N-1)
      NM1=N-1
      YA(1)=DATA(1)
      YB(NM1)=DATA(1)
      DO 20 I=2,NM1
      YA(I)=DATA(I)
   20 YB(I-1)=DATA(I)
```

```
C
C     WE WISH TO FIND THE MINIMUM VALUE OF THE AKAIKE FPE AND THE
C          COEFFICIENTS ASSOCIATED WITH THIS MINIMUM, UNLESS THE NUMBER
C          OF FILTER COEFFICIENTS HAS BEFN SPECIFIED.
C
      IF(MEMN.EQ.0)GO TO 25
      WRITE(6,145) MEMN,MEMX,DT
      GO TO 30
   25 WRITE(6,146) MEMX,DT
      WRITE(6,147)
      WRITE(6,148)
   30 DO 75 M=1,N
      MA=M-1
      MB=N-M
      IF(M.EQ.1) GO TO 45
      DO 35 I=1,MA
   35 AB(I)=AA(I)
      DO 40 I=1,MB
      YA(I)=YA(I)-AB(M-1)*YB(I)
   40 YB(I)=YB(I+1)-AB(M-1)*YA(I+1)
   45 DOM=0.0
      DEN=0.0
      DO 50 I=1,MB
      DOM=DOM+YA(I)*YB(I)
   50 DEN=DEN+YA(I)*YA(I)+YB(I)*YB(I)
      AA(M)=2.0*DOM/DEN
      PM=PM*(1.0-AA(M)*AA(M))
      IF(M.EQ.1)GO TO 70
      DO 55 I=1,MA
   55 AA(I)=AB(I)-AA(M)*AB(M-I)
      IF(MEMN-M) 57,61,75
C
C     THE RESIDUAL SUM OF SQUARES IS NOW CALCULATED IF THE AKAIKE FPE IS
C          TO BE DETERMINED BY THE SUBROUTINE.
C
   57 CALL SMSQR(DATA,N,AA,M,SM)
      XFPE=SM*FLOAT(N+M+1)/FLOAT(N-M-1)
      IF(XFPE.LT.FPEM) GO TO 60
      IS=IS+1
      GO TO 73
C
C     STORE THE LATEST SET OF VALUES.
C
   60 FPEM=XFPE
   61 PN=PM
      MM=M
      DO 62 I=1,M
   62 CA(I)=AA(I)
      IF(M.EQ.MEMN)GO TO 80
      IS=0
      GO TO 73
C
C     CALCULATE THE INITIAL VALUES.
C
   70 IF(MEMN.GT.0)GO TO 71
      CALL SMSQR(DATA,N,AA,M,SM)
      FPEM=SM*FLOAT(N+2)/FLOAT(N-2)
   71 MM=1
      PN=PM
      CA(1)=AA(1)
```

```
      IF(MEMN.EQ.M)GO TO 80
      IF(MEMN.GT.0)GO TO 75
      XFPE=FPEM
   73 WRITE(6,149) M,XFPE,PM
      IF(IS.GT.MT) GO TO 80
   75 CONTINUE
C
C     NOW WE CALCULATE THE FREQUENCY AND POWER VALUES.
C     A MINIMUM VALUE OF .000011 IS ASSIGNED TO THE POWER (AFTER
C          PRINTING) IN ORDER TO SIMPLIFY THE CHOICE OF PLOTTING
C          PARAMETERS.
C
   80 IF(MEMN.GT.0)FPEM=0.0
      WRITE(6,150) MM,FPEM,PN
      WRITE(6,151) (CA(I),I=1,MM)
      T2PI=DT*6.283185307
      BX=.000011
      RNZ=2.0*DT*FLOAT(N)*FLOAT(MEMX)
      PT=PN*DT
      NZP=N+1
      RFPE=FPEM/FPEO
      WRITE(6,152) FPEO,FPEM,RFPE
      WRITE(6,153)
      DO 90 I=1,NZP
      FREQ(I)=FLOAT(I-1)/RNZ
      ARG=T2PI*FREQ(I)
      SREAL=1.0
      SIMAG=0.0
      DO 85 J=1,MM
      SREAL=SREAL-CA(J)*COS(ARG*FLOAT(J))
   85 SIMAG=SIMAG+CA(J)*SIN(ARG*FLOAT(J))
      P(I)=PT/(SREAL*SREAL+SIMAG*SIMAG)
      WRITE(6,154) FREQ(I),P(I)
      IF(P(I).LT.BX)P(I)=BX
   90 CONTINUE
      RETURN
      END
```

Table B.10. Subroutine SMSQR, as described in Chapter 5.

```
      SUBROUTINE SMSQR(X,N,A,M,SM)
C     ***********************************************************************
C     THIS PROGRAM CALCULATES A RESIDUAL SUM OF SQUARES NEEDED TO
C          COMPUTE THE AKAIKE FPE.
C     X = DATA ARRAY.
C     N = NUMBER OF DATA VALUES.
C     A = COEFFICIENT ARRAY.
C     M = NUMBER OF COEFFICIENT VALUES.
C     SM = RESIDUAL SUM OF SQUARES.
C     ***********************************************************************
C
      DIMENSION X(N),A(M)
      SUM=0.0
      DO 30 I=1,N
      SUN=0.0
      DO 10 J=1,M
      IF(J.GE.I)GO TO 20
   10 SUN=SUN+A(J)*X(I-J)
   20 CONTINUE
   30 SUM=SUM+(X(I)-SUN)**2
      SM=SUM/FLOAT(N)
      RETURN
      END
```

Table B.11. Subroutine PLTDAT, as described in Chapter 6.

```
      SUBROUTINE PLTDAT(DATA,N)
C     ******************************************************************
C     THE RAW DATA IS STORED IN ARRAY DATA.
C     ARRAY YRS IS CALCULATED AND CONTAINS VALUES FOR THE ABSCISSA AXIS.
C     ALPHA CONTAINS THE TITLE AND AXIS LABELS.
C     Z PLOTTING FACILITY NOT AVAILABLE.
C     PARAMETERS INPUT IN THIS SUBPROGRAM (FROM THE FILE ASSIGNED TO
C          UNIT 7):
C        YR1 = FIRST YEAR TO BE PLOTTED (WITH DECIMAL PERIOD IF FIRST
C          MONTH IS NOT JANUARY).
C        YRIN = INCREMENT OF TIME FOR EACH DATA POINT.
C        NF = AS IN CSLIB (COMPUTING SCIENCE LIBRARY WRITE-UP FOR CGPL).
C        KA AND KB AS IN CSLIB EXCEPT FOR KA=KB=1, WHERE -1 ROTATES THE
C          NUMBERS 90 DEGREES W.R.T. THE AXIS, FOR ARITHMETIC AXIS ONLY.
C        KC AS IN CSLIB FOR KC = 3 TO 6 (ALSO SEE COMMENTS IN OTHER
C          PLOTTING SUBROUTINES).
C          FOR KC = 1 OR 2 SYMBOL HEIGHT DETERMINED BY HT,
C          FOR KC = 7 SYMBOLS OF HEIGHT HT ARE PLOTTED WITH NO LINE,
C            ACCORDING TO SYD AND SYS.
C          FOR KC = 8 SYMBOLS OF HEIGHT HT ARE PLOTTED AND JOINED BY
C            STRAIGHT LINE SEGMENTS ACCORDING TO SYD AND SYS.
C        HA,HB,HC AS IN CSLIB.
C        VA,VB,VC AS IN CSLIB.
C        HT HEIGHT OF PLOTTED SYMBOLS IN DECIMAL INCHES
C          (DUMMY VARIABLE USED WHEN KC = 3 TO 6).
C        SYD DELAY (IN DECIMAL INCHES) BEFORE FIRST SYMBOL IS PLOTTED
C          ON THE LINE.
C        SYS SEPARATION (IN DECIMAL INCHES) BETWEEN SYMBOLS PLOTTED
C          ON THE LINE.
C        NOTE: SYD AND SYS MAY BE DUMMY VARIABLES WHEN KC = 1 TO 6.
C     ******************************************************************
  600 FORMAT(F9.4,F6.4)
  601 FORMAT(20A4)
  602 FORMAT(4I5,6F10.3)
  603 FORMAT(3F10.3)
C     ******************************************************************
C
      DIMENSION DATA(N),YRS(4096),ALPHA(20)
      READ(7,600)YR1,YRIN
      DO 10 I=1,N
   10 YRS(I)=YR1+(I-1)*YRIN
      READ(7,601)(ALPHA(I),I=1,20)
      READ(7,602)NF,KA,KB,KC,HA,HB,HC,VA,VB,VC
      READ(7,603)HT,SYD,SYS
C
C     CALL THE PLOTTING SUBROUTINE.
C
      CALL PLTG(YRS,DATA,DATA,N,NF,KA,KB,KC,KC,HA,HB,HC,VA,VB,VC,ALPHA,6
     *,HT,SYD,SYS,1)
      RETURN
      END
```

Table B.12. Subroutine PLTFFT, as described in Chapter 6.

```
      SUBROUTINE PLTFFT(N,M)
C     ************************************************************************
C     THE PARAMETERS TO BE READ IN THIS SUBROUTINE (FROM UNIT 7) ARE THE
C          SAME AS IN PLTDAT.
C     IF NO PLOT IS WANTED AT THIS TIME BUT THE INFORMATION IS TO BE
C          STORED, SET KC NEGATIVE AND ASSIGN A FILE TO UNIT 1.
C     ************************************************************************
  610 FORMAT(20A4)
  611 FORMAT(4I5,6F10.3)
  612 FORMAT(3F10.3)
  613 FORMAT(8F10.5)
C     ************************************************************************
C
      DIMENSION ALPHA(20)
      COMMON /OUTPLT/ FREQ(2050),POW(2050),SOW(2050)
      IF(M.EQ.2)GO TO 20
C
C     PLOT THE UNSMOOTHED POWER SPECTRUM.
C
      READ(7,610) (ALPHA(I),I=1,20)
      READ(7,611)NF,KA,KB,KC,HA,HB,HC,VA,VB,VC
      READ(7,612)HT,SYD,SYS
      IF(KC.GT.0)GO TO 10
      WRITE(1,613) (FREQ(I),POW(I),I=1,N)
      GO TO 20
   10 CALL PLTG(FREQ,POW,POW,N,NF,KA,KB,KC,KC,HA,HB,HC,VA,VB,VC,ALPHA,6,
     *HT,SYD,SYS,1)
      IF(M.EQ.1)GO TO 40
C
C     PLOT THE SMOOTHED POWER SPECTRUM.
C
   20 READ(7,610) (ALPHA(I),I=1,20)
      READ(7,611)NF,KA,KB,KC,HA,HB,HC,VA,VB,VC
      READ(7,612)HT,SYD,SYS
      IF(KC.GT.0)GO TO 30
      WRITE(1,613) (FREQ(I),SOW(I),I=1,N)
      RETURN
   30 CALL PLTG(FREQ,SOW,SOW,N,NF,KA,KB,KC,KC,HA,HB,HC,VA,VB,VC,ALPHA,6,
     *HT,SYD,SYS,1)
   40 RETURN
      END
```

Table B.13. Subroutine PLTMEM, as described in Chapter 6.

```
      SUBROUTINE PLTMEM(N)
C     ***********************************************************************
C     THE PARAMETERS TO BE READ IN THIS SUBROUTINE (FROM UNIT 7) ARE THE
C          SAME AS IN PLTDAT.
C     IF NO PLOT IS WANTED AT THIS TIME BUT THE INFORMATION IS TO BE
C          STORED, SET KC NEGATIVE AND ASSIGN A FILE TO UNIT 2.
C     ***********************************************************************
  620 FORMAT(20A4)
  621 FORMAT(4I5,6F10.3)
  622 FORMAT(3F10.3)
  623 FORMAT(8F10.5)
C     ***********************************************************************
C
      DIMENSION ALPHA(20)
      COMMON /MEMOUT/ FREQ(4097),P(4097)
      READ(7,620) (ALPHA(I),I=1,20)
      READ(7,621)NF,KA,KB,KC,HA,HB,HC,VA,VB,VC
      READ(7,622)HT,SYD,SYS
      IF(KC.LE.0)GO TO 20
      CALL PLTG(FREQ,P,P,N,NF,KA,KB,KC,KC,HA,HB,HC,VA,VB,VC,ALPHA,6,HT,S
     *YD,SYS,1)
      RETURN
   20 WRITE(2,623) (FREQ(I),P(I),I=1,N)
      RETURN
      END
```

Table B.14. Notch filter program, as described in Chapter 6.

```
C      NOTCH FILTER
C      ***********************************************************************
C      THIS PROGRAM PERFORMS A FILTERING PROCESS ON THE DATA USING A
C           NOTCH FILTER TO REMOVE A PARTICULAR FREQUENCY.
C      IF REQUIRED, THE PROGRAM WILL CALCULATE THE FREQUENCY RESPONSE
C           FUNCTION; IT CAN ALSO PLOT THIS FUNCTION.
C      ZA = THE NEIGHBOURHOOD OF THE UNIT CIRCLE DESIRED.
C      M = NUMBER OF TIMES THAT THE FILTER IS APPLIED; ONE APPLICATION
C           INVOLVES A FOWARD AND A REVERSE PASS OVER THE DATA.
C      ZB = THE ANGLE ON THE UNIT CIRCLE OF THE UNWANTED FREQUENCY, IN
C           RADIANS.
C      NFR = 0, NO CALCULATION OF THE FREQUENCY RESPONSE FUNCTION.
C          = NUMBER OF FREQUENCY VALUES REQUIRED IN THE FREQUENCY
C             RESPONSE FUNCTION.
C          (POSITIVE) RESULTS ARE PRINTED AND PLOTTED.
C          (NEGATIVE) RESULTS ARE PRINTED ONLY.
C      THE REMAINING INPUT DATA IS THE SAME AS FOR THE MAIN PROGRAM OF
C           ANALYSIS.
C      THE PARAMETERS TO BE READ FOR PLOTTING (FROM UNIT 7) ARE THE SAME
C           AS DESCRIBED IN THE MAIN ANALYSIS PROGRAM.
C
C      THIS PROGRAM CAN BE RUN IMMEDIATELY BEFORE THE ANALYSIS PROGRAM IF
C           UNIT 8 IS ASSIGNED TO THE FILE FROM WHICH THE ANALYSIS
C           PROGRAM WILL OBTAIN ITS INFORMATION.
C      ***********************************************************************
   700 FORMAT(F7.5,I3,F10.7,I5)
   701 FORMAT(13I5,F5.1,F5.3)
   702 FORMAT(1H1,'THE COEFFICIENTS FOR THE NOTCH FILTER ARE:',3F10.7)
   703 FORMAT(1H-,12X,'FREQUENCY',11X,'RESPONSE',/)
   704 FORMAT(1H ,10X,F10.5,10X,F10.6)
   705 FORMAT(20A4)
   706 FORMAT(4I5,6F10.3)
   707 FORMAT(3F10.3)
   708 FORMAT(15A4)
   709 FORMAT(1H-,'FOR THE FREQUENCY = ',F10.5,', THE MODULUS OF THE FILT
      *ER IS ',E12.5,' FOR EACH OF THE',I3,' CASCADE(S).')
C      ***********************************************************************
C
       DIMENSION X(4196),Y(4196),F(4097),V(4097),ALPHA(20),FOR1(15),FOR2(
      *15)
       COMPLEX YA,YB,YC,CMPLX,CABS
       READ(5,700) ZA,M,ZB,NFR
       A=1.0/(ZA**2)
       B=2.0*COS(ZB)
       C=B/ZA
       READ(5,701) II,N,JA,JB,JC,JD,JE,JF,JG,JH,JI,JJ,JK,AA,DT
       WRITE(6,702) A,B,C
       PI2T=2.0*3.1415927*DT
       IF(NFR.EQ.0) GO TO 10
       NUM=IABS(NFR)
       MUN=NUM+1
       DEN=FLOAT(2*NUM)
       WRITE(6,703)
       DO 5 I=1,MUN
       F(I)=FLOAT(I-1)/DEN
```

```
      ZC=PI2T*F(I)
      ZD=2.0*ZC
      YA=CMPLX(COS(ZC),SIN(ZC))
      YB=CMPLX(COS(ZD),SIN(ZD))
      YC=(A*(1.0-B*YA+YB))/(1.0-C*YA+A*YB)
      V(I)=CABS(YC)
      WRITE(6,704)F(I),V(I)
    5 CONTINUE
      IF(NFR.LE.0)GO TO 10
      READ(7,705)(ALPHA(I),I=1,20)
      READ(7,706)NF,KA,KB,KC,HA,HB,HC,VA,VB,VC
      READ(7,707)HT,SYD,SYS
      CALL PLTG(F,V,V,MUN,NF,KA,KB,KC,KC,HA,HB,HC,VA,VB,VC,ALPHA,6,HT,SY
     *D,SYS,1)
   10 IF(II.EQ.0)GO TO 15
      READ(5,708)FOR1
   15 READ(5,708)FOR2
      II=II+100
      NO=N+100
      IF(II.EQ.100)GO TO 20
      READ(5,FOR1)(X(L),L=101,II)
   20 IM=II+1
      READ(5,FOR2)(X(L),L=IM,NO)
      N1=N+101
      N4=N+196
      N6=N+198
      N7=N+199
      N8=N+200
      N9=N+201
      DO 25 I=1,100
      X(I)=0.0
   25 Y(I)=0.0
      DO 30 I=N1,N8
      X(I)=0.0
   30 Y(I)=0.0
      DO 60 L=1,M
      DO 40 I=3,N6
   40 Y(I)=A*(X(I)-B*X(I-1)+X(I-2)-Y(I-2))+C*Y(I-1)
      DO 50 I=1,N4
   50 X(N7-I)=A*(Y(N7-I)-B*Y(N8-I)+Y(N9-I)-X(N9-I))+C*X(N8-I)
   60 CONTINUE
      II=0
      WRITE(8,701)II,N,JA,JB,JC,JD,JE,JF,JG,JH,JI,JJ,JK,AA,DT
      WRITE(8,708)FOR2
      WRITE(8,FOR2)(X(I),I=101,NO)
      FREQ=ZB/PI2T
      ZD=2.0*ZB
      YA=CMPLX(COS(ZB),SIN(ZB))
      YB=CMPLX(COS(ZD),SIN(ZD))
      YC=(A*(1.0-B*YA+YB))/(1.0-C*YA+A*YB)
      YC=YC*YC
      AMP=CABS(YC)
      WRITE(6,709)FREQ,AMP,M
      STOP
      END
```

Table B.15. Sample print-out of data and FFT output with no smoothing applied.

JOB NUMBER 1 WITH 68 VALUES USED OF WHICH 68 VALUES ARE PLOTTED AND/OR PRINTED; THE SIGN FOR THE FFT IS -1.0

RAW DATA MEAN = -11.407 RAW DATA VARIANCE = 14.681 RAW DATA STANDARD DEVIATION = 3.832

POINT NO.	1	2	3	4	5	6	7	8	9	10	11	12
VALUE	-13.30	-17.10	-14.60	-15.10	-8.40	-9.60	-10.10	-15.10	-11.60	-5.30	-12.30	-17.90
POINT NO.	13	14	15	16	17	18	19	20	21	22	23	24
VALUE	-10.90	-15.20	-11.70	-6.20	-10.80	-13.20	-11.10	-19.90	-7.00	-8.30	-10.20	-8.40
POINT NO.	25	26	27	28	29	30	31	32	33	34	35	36
VALUE	-18.10	-13.30	-10.60	-16.80	-5.40	-12.80	-11.80	-6.60	-15.30	-10.80	-15.00	-4.00
POINT NO.	37	38	39	40	41	42	43	44	45	46	47	48
VALUE	-9.20	-16.80	-13.90	-9.90	-9.60	-4.70	-17.30	-12.80	-5.30	-7.60	-5.30	-19.30
POINT NO.	49	50	51	52	53	54	55	56	57	58	59	60
VALUE	-6.40	-8.20	-9.90	-11.90	-12.90	-11.80	-9.10	-13.30	-10.50	-12.30	-18.90	-7.40
POINT NO.	61	62	63	64	65	66	67	68				
VALUE	-9.80	-10.00	-11.70	-11.30	-9.40	-15.80	-7.00	-12.60				

FOR THE F.F.T., THE MEAN = -0.000049 TOTAL VARIANCE = 7.79876 STANDARD DEVIATION = 2.7926
AFTER APPLICATION OF A 0 POINT TAPER.

NO SMOOTHING HAS BEEN APPLIED

FREQUENCY	COSINE	SINE	PHASE	POWER	AMPL. SQ.
0.0	-0.43213E-06	0.0	-180.000	0.37348E-12	0.10965E-10
0.00781	-0.68575E-01	-0.22263E-01	-162.013	0.10396E-01	0.30523E+00
0.01563	-0.41897E-01	0.57028E-01	126.304	0.10015E-01	0.29404E+00
0.02344	0.18676E-01	0.55402E-02	16.523	0.75899E-03	0.22284E-01
0.03125	-0.44582E-01	-0.25104E-01	-150.616	0.52355E-02	0.15372E+00
0.03906	-0.30985E-01	0.41802E-01	126.548	0.54149E-02	0.15898E+00
0.04688	0.52017E-02	-0.19944E-01	-75.382	0.84966E-03	0.24946E-01
0.05469	-0.60963E-01	0.95808E-02	171.069	0.76167E-02	0.22363E+00
0.06250	0.13613E-01	0.15044E-01	47.860	0.82327E-03	0.24171E-01
0.07031	-0.49760E-01	-0.43280E-01	-138.984	0.86986E-02	0.25539E+00

0.07813	-0.65576E-01	0.26296E-01	158.149	0.99834E-02	0.29311E+00
0.08594	-0.43514E-01	0.31128E-01	144.421	0.57248E-02	0.16808E+00
0.09375	-0.17476E-01	0.39793E-01	113.710	0.37778E-02	0.11092E+00
0.10156	-0.28015E-01	0.49413E-02	169.997	0.16185E-02	0.47520E-01
0.10938	-0.39733E-01	0.39755E-01	134.984	0.63182E-02	0.18550E+00
0.11719	-0.33088E-01	0.28242E-01	139.518	0.37848E-02	0.11112E+00
0.12500	-0.30109E-01	0.87990E-01	108.890	0.17298E-01	0.50786E+00
0.13281	0.58703E-01	0.51972E-01	41.519	0.12294E-01	0.36096E+00
0.14063	0.40904E-01	0.18010E-02	2.521	0.33528E-02	0.98439E-01
0.14844	0.30947E-01	-0.70475E-01	-66.293	0.11849E-01	0.34788E+00
0.15625	-0.11379E+00	-0.46735E-01	-157.672	0.30265E-01	0.88858E+00
0.16406	-0.81621E-01	0.64612E-01	141.634	0.21673E-01	0.63633E+00
0.17188	-0.53185E-01	0.10201E+00	117.536	0.26470E-01	0.77717E+00
0.17969	0.59425E-01	0.78249E-01	52.786	0.19308E-01	0.56690E+00
0.18750	-0.39684E-01	0.20275E-01	152.938	0.39718E-02	0.11661E+00
0.19531	0.69494E-01	0.15034E+00	65.191	0.54863E-01	0.16108E+01
0.20313	0.12353E+00	-0.81098E-01	-33.286	0.43671E-01	0.12822E+01
0.21094	-0.10310E+00	-0.16191E-01	-171.074	0.21782E-01	0.63951E+00
0.21875	0.45453E-01	0.13732E+00	71.686	0.41847E-01	0.12286E+01
0.22656	0.12803E+00	-0.35156E-01	-15.355	0.35253E-01	0.10350E+01
0.23438	-0.16131E-01	-0.61103E-01	-104.789	0.79877E-02	0.23452E+00
0.24219	0.93478E-02	0.35991E-01	75.441	0.27655E-02	0.81196E-01
0.25000	0.74415E-01	-0.43921E-01	-30.550	0.14933E-01	0.43844E+00
0.25781	-0.44719E-01	-0.69610E-01	-122.718	0.13691E-01	0.40196E+00
0.26563	-0.18044E-02	0.79995E-02	102.711	0.13450E-03	0.39488E-02
0.27344	-0.53728E-01	-0.54395E-01	-134.647	0.11691E-01	0.34325E+00
0.28125	-0.19829E-01	0.70905E-01	105.624	0.10841E-01	0.31830E+00
0.28906	0.12616E-01	-0.61459E-01	-78.400	0.78728E-02	0.23114E+00
0.29688	-0.81954E-01	0.26672E-01	161.972	0.14856E-01	0.43616E+00
0.30469	-0.31213E-01	0.37462E-01	129.801	0.47553E-02	0.13962E+00
0.31250	0.11153E-01	0.94392E-01	83.262	0.18069E-01	0.53049E+00
0.32031	0.45158E-01	-0.20725E-01	-24.653	0.49375E-02	0.14496E+00
0.32813	-0.20702E-01	0.52493E-01	111.523	0.63683E-02	0.18697E+00
0.33594	0.61198E-01	0.18717E-01	17.006	0.81911E-02	0.24049E+00
0.34375	0.47298E-01	0.11730E-01	13.928	0.47495E-02	0.13944E+00
0.35156	0.69225E-01	-0.67775E-01	-44.394	0.18771E-01	0.55112E+00
0.35938	-0.29353E-01	-0.83524E-01	-109.363	0.15676E-01	0.46024E+00
0.36719	-0.57048E-01	-0.41815E-01	-143.759	0.10006E-01	0.29378E+00
0.37500	-0.10805E+00	-0.32264E-01	-163.374	0.25431E-01	0.74665E+00
0.38281	-0.12650E+00	0.17297E+00	126.179	0.91841E-01	0.26965E+01
0.39063	0.20380E+00	0.10500E+00	27.258	0.10512E+00	0.30862E+01
0.39844	-0.15100E-02	-0.14426E+00	-90.600	0.41625E-01	0.12221E+01
0.40625	-0.47250E-01	0.25737E-01	151.423	0.57898E-02	0.16999E+00
0.41406	-0.46454E-01	0.21365E-01	155.302	0.52289E-02	0.15352E+00
0.42188	0.36519E-01	0.55977E-01	56.879	0.89341E-02	0.26231E+00
0.42969	-0.18607E-01	0.13308E-01	144.428	0.10466E-02	0.30730E-01
0.43750	0.82141E-01	0.20902E-01	14.277	0.14368E-01	0.42185E+00
0.44531	-0.74903E-01	-0.44361E-01	-149.364	0.15157E-01	0.44500E+00

0.45313	0.57232E-01	0.11358E+00	63.256	0.32350E-01	0.94980E+00
0.46094	0.47872E-01	-0.86690E-01	-61.092	0.19614E-01	0.57586E+00
0.46875	-0.60037E-01	0.15704E-02	178.502	0.72139E-02	0.21180E+00
0.47656	-0.14940E-01	0.66992E-01	102.572	0.94224E-02	0.27664E+00
0.48438	0.62521E-01	0.26302E-01	22.816	0.92012E-02	0.27015E+00
0.49219	0.94178E-02	0.17485E-01	61.693	0.78888E-03	0.23161E-01
0.50000	0.75254E-01	0.0	0.0	0.11326E-01	0.33254E+00
THE SUMS ARE				0.10057E+01	0.29526E+02

Table B.16. Sample print-out of FFT output with smoothing applied to the frequency and phase values.

JOB NUMBER 2 WITH 68 VALUES USED OF WHICH 0 VALUES ARE PLOTTED AND/OR PRINTED; THE SIGN FOR THE FFT IS -1.0

RAW DATA MEAN = -11.407 RAW DATA VARIANCE = 14.681 RAW DATA STANDARD DEVIATION = 3.832

FOR THE F.F.T., THE MEAN = -0.000049 TOTAL VARIANCE = 7.79876 STANDARD DEVIATION = 2.7926
AFTER APPLICATION OF A 0 POINT TAPER.

ALL SMOOTHING APPLIED USING A 3 POINT DANIELL (BOXCAR) FILTER

FREQUENCY	COSINE	SINE	PHASE	UNSMOOTHED POWER	UNSMOOTHED AMPL. SQ.	WEIGHTED FREQUENCY	SMOOTHED PHASE
0.0	-0.43213E-06	0.0	-180.000	0.37348E-12	0.10965E-10	0.0	-180.000
0.00781	-0.68575E-01	-0.22263E-01	-162.013	0.10396E-01	0.30523E+00	0.01165	162.532
0.01563	-0.41897E-01	0.57028E-01	126.304	0.10015E-01	0.29404E+00	0.01207	156.295
0.02344	0.18676E-01	0.55402E-02	16.523	0.75899E-03	0.22284E-01	0.02111	151.078
0.03125	-0.44582E-01	-0.25104E-01	-150.616	0.52355E-02	0.15372E+00	0.03444	158.651
0.03906	-0.30985E-01	0.41802E-01	126.548	0.54149E-02	0.15898E+00	0.03608	-177.358
0.04688	0.52017E-02	-0.19944E-01	-75.382	0.84966E-03	0.24946E-01	0.04811	160.079
0.05469	-0.60963E-01	0.95808E-02	171.069	0.76167E-02	0.22363E+00	0.05467	173.663
0.06250	0.13613E-01	0.15044E-01	47.860	0.82327E-03	0.24171E-01	0.06299	-169.126
0.07031	-0.49760E-01	-0.43280E-01	-138.984	0.86986E-02	0.25539E+00	0.07398	-178.907
0.07813	-0.65576E-01	0.26296E-01	158.149	0.99834E-02	0.29311E+00	0.07717	174.912
0.08594	-0.43514E-01	0.31128E-01	144.421	0.57248E-02	0.16808E+00	0.08345	142.471
0.09375	-0.17476E-01	0.39793E-01	113.710	0.37778E-02	0.11092E+00	0.09087	139.558
0.10156	-0.28015E-01	0.49413E-02	169.997	0.16185E-02	0.47520E-01	0.10326	135.248
0.10938	-0.39733E-01	0.39755E-01	134.984	0.63182E-02	0.18550E+00	0.11082	144.120
0.11719	-0.33088E-01	0.28242E-01	139.518	0.37848E-02	0.11112E+00	0.12032	123.419
0.12500	-0.30109E-01	0.87990E-01	108.890	0.17298E-01	0.50786E+00	0.12699	91.530
0.13281	0.58703E-01	0.51972E-01	41.519	0.12294E-01	0.36096E+00	0.12951	63.884
0.14063	0.40904E-01	0.18010E-02	2.521	0.33528E-02	0.98439E-01	0.14050	-7.290
0.14844	0.30947E-01	-0.70475E-01	-66.293	0.11849E-01	0.34788E+00	0.15306	-109.971
0.15625	-0.11379E+00	-0.46735E-01	-157.672	0.30265E-01	0.88858E+00	0.15745	-162.265
0.16406	-0.81621E-01	0.64612E-01	141.634	0.21673E-01	0.63633E+00	0.16368	154.253
0.17188	-0.53185E-01	0.10201E+00	117.536	0.26470E-01	0.77717E+00	0.17160	107.110
0.17969	0.59425E-01	0.78249E-01	52.786	0.19308E-01	0.56690E+00	0.17615	99.469
0.18750	-0.39684E-01	0.20275E-01	152.938	0.39718E-02	0.11661E+00	0.19105	70.274
0.19531	0.69494E-01	0.15034E+00	65.191	0.54863E-01	0.16108E+01	0.19834	30.276
0.20313	0.12353E+00	-0.81098E-01	-33.286	0.43671E-01	0.12822E+01	0.20098	30.538
0.21094	-0.10310E+00	-0.16191E-01	-171.074	0.21782E-01	0.63951E+00	0.21080	31.285

0.21875	0.45453E-01	0.13732E+00	71.686	0.41847E-01	0.12286E+01	0.21981	50.695
0.22656	0.12803E+00	-0.35156E-01	-15.355	0.35253E-01	0.10350E+01	0.22345	14.627
0.23438	-0.16131E-01	-0.61103E-01	-104.789	0.79877E-02	0.23452E+00	0.22886	-26.431
0.24219	0.93478E-02	0.35991E-01	75.441	0.27655E-02	0.81196E-01	0.24430	-45.588
0.25000	0.74415E-01	-0.43921E-01	-30.550	0.14933E-01	0.43844E+00	0.25272	-63.274
0.25781	-0.44719E-01	-0.69610E-01	-122.718	0.13691E-01	0.40196E+00	0.25379	-75.196
0.26563	-0.18044E-02	0.79995E-02	102.711	0.13450E-03	0.39488E-02	0.26501	-130.834
0.27344	-0.53728E-01	-0.54395E-01	-134.647	0.11691E-01	0.34325E+00	0.27713	161.984
0.28125	-0.19829E-01	0.70905E-01	105.624	0.10841E-01	0.31830E+00	0.28027	-143.588
0.28906	0.12616E-01	-0.61459E-01	-78.400	0.78728E-02	0.23114E+00	0.29000	157.949
0.29688	-0.81954E-01	0.26672E-01	161.972	0.14856E-01	0.43616E+00	0.29599	178.476
0.30469	-0.31213E-01	0.37462E-01	129.801	0.47553E-02	0.13962E+00	0.30535	122.762
0.31250	0.11153E-01	0.94392E-01	83.262	0.18069E-01	0.53049E+00	0.31255	77.274
0.32031	0.45158E-01	-0.20725E-01	-24.653	0.49375E-02	0.14496E+00	0.31720	74.239
0.32813	-0.20702E-01	0.52493E-01	111.523	0.63683E-02	0.18697E+00	0.32943	30.516
0.33594	0.61198E-01	0.18717E-01	17.006	0.81911E-02	0.24049E+00	0.33528	43.371
0.34375	0.47298E-01	0.11730E-01	13.928	0.47495E-02	0.13944E+00	0.34636	-11.862
0.35156	0.69225E-01	-0.67775E-01	-44.394	0.18771E-01	0.55112E+00	0.35374	-58.012
0.35938	-0.29353E-01	-0.83524E-01	-109.363	0.15676E-01	0.46024E+00	0.35783	-95.083
0.36719	-0.57048E-01	-0.41815E-01	-143.759	0.10006E-01	0.29378E+00	0.36868	-140.975
0.37500	-0.10805E+00	-0.32264E-01	-163.374	0.25431E-01	0.74665E+00	0.38002	161.266
0.38281	-0.12650E+00	0.17297E+00	126.179	0.91841E-01	0.26965E+01	0.38561	97.133
0.39063	0.20380E+00	0.10500E+00	27.258	0.10512E+00	0.30862E+01	0.38898	60.454
0.39844	-0.15100E-02	-0.14426E+00	-90.600	0.41625E-01	0.12221E+01	0.39335	-4.985
0.40625	-0.47250E-01	0.25737E-01	151.423	0.57898E-02	0.16999E+00	0.40085	-134.422
0.41406	-0.46454E-01	0.21365E-01	155.302	0.52289E-02	0.15352E+00	0.41529	119.020
0.42188	0.36519E-01	0.55977E-01	56.879	0.89341E-02	0.26231E+00	0.41973	107.477
0.42969	-0.18607E-01	0.13308E-01	144.428	0.10466E-02	0.30730E-01	0.43143	42.031
0.43750	0.82141E-01	0.20902E-01	14.277	0.14368E-01	0.42185E+00	0.44111	-138.238
0.44531	-0.74903E-01	-0.44361E-01	-149.364	0.15157E-01	0.44500E+00	0.44758	54.420
0.45313	0.57232E-01	0.11358E+00	63.256	0.32350E-01	0.94980E+00	0.45364	-30.056
0.46094	0.47872E-01	-0.86690E-01	-61.092	0.19614E-01	0.57586E+00	0.45762	32.269
0.46875	-0.60037E-01	0.15704E-02	178.502	0.72139E-02	0.21180E+00	0.46655	-146.226
0.47656	-0.14940E-01	0.66992E-01	102.572	0.94224E-02	0.27664E+00	0.47716	97.481
0.48438	0.62521E-01	0.26302E-01	22.816	0.92012E-02	0.27015E+00	0.48090	62.773
0.49219	0.94178E-02	0.17485E-01	61.693	0.78888E-03	0.23161E-01	0.49297	16.567
0.50000	0.75254E-01	0.0	0.0	0.11326E-01	0.33254E+00	0.50000	0.0

THE SUMS ARE 0.10057E+01 0.29526E+02

ALSO NOTE THAT THE FIRST AND LAST 1 POINT(S) OF THE SMOOTHED PHASE
AND WEIGHTED FREQUENCY COLUMNS HAVE NOT BEEN SMOOTHED

Table B.17. Sample print-out of FFT output with smoothing applied to the frequency, phase, power and amplitude squared.

JOB NUMBER 3 WITH 68 VALUES USED OF WHICH 0 VALUFS ARE PLOTTED AND/OR PRINTED; THE SIGN FOR THE FFT IS -1.0

RAW DATA MEAN = -11.407 RAW DATA VARIANCE = 14.681 RAW DATA STANDARD DEVIATION = 3.832

FOR THE F.F.T., THE MEAN = -0.000049 TOTAL VARIANCE = 7.79876 STANDARD DEVIATION = 2.7926
AFTER APPLICATION OF A 0 POINT TAPER.

ALL SMOOTHING APPLIED USING A 3 POINT DANIELL (BOXCAR) FILTER

FREQUENCY	COSINE	SINE	PHASE	UNSMOOTHED POWER	SMOOTHED POWER	SMOOTHED AMPL. SQ.	WEIGHTED FREQUENCY	SMOOTHED PHASE
0.0	-0.43213E-06	0.0	-180.000	0.37348E-12	0.37348E-12	0.10965E-10	0.0	-180.000
0.00781	-0.68575E-01	-0.22263E-01	-162.013	0.10396E-01	0.68037E-02	0.19976E+00	0.01165	162.532
0.01563	-0.41897E-01	0.57028E-01	126.304	0.10015E-01	0.70567E-02	0.20719E+00	0.01207	156.295
0.02344	0.18676E-01	0.55402E-02	16.523	0.75899E-03	0.53365E-02	0.15668E+00	0.02111	151.078
0.03125	-0.44582E-01	-0.25104E-01	-150.616	0.52355E-02	0.38032E-02	0.11166E+00	0.03444	158.651
0.03906	-0.30985E-01	0.41802E-01	126.548	0.54149E-02	0.38334E-02	0.11255E+00	0.03608	-177.358
0.04688	0.52017E-02	-0.19944E-01	-75.382	0.84966E-03	0.46271E-02	0.13585E+00	0.04811	160.079
0.05469	-0.60963E-01	0.95808E-02	171.069	0.76167E-02	0.30965E-02	0.90914E-01	0.05467	173.663
0.06250	0.13613E-01	0.15044E-01	47.860	0.82327E-03	0.57128E-02	0.16773E+00	0.06299	-169.126
0.07031	-0.49760E-01	-0.43280E-01	-138.984	0.86986E-02	0.65017E-02	0.19089E+00	0.07398	-178.907
0.07813	-0.65576E-01	0.26296E-01	158.149	0.99834E-02	0.81356E-02	0.23886E+00	0.07717	174.912
0.08594	-0.43514E-01	0.31128E-01	144.421	0.57248E-02	0.64953E-02	0.19070E+00	0.08345	142.471
0.09375	-0.17476E-01	0.39793E-01	113.710	0.37778E-02	0.37070E-02	0.10884E+00	0.09087	139.558
0.10156	-0.28015E-01	0.49413E-02	169.997	0.16185E-02	0.39049E-02	0.11465E+00	0.10326	135.248
0.10938	-0.39733E-01	0.39755E-01	134.984	0.63182E-02	0.39072E-02	0.11471E+00	0.11082	144.120
0.11719	-0.33088E-01	0.28242E-01	139.518	0.37848E-02	0.91335E-02	0.26816E+00	0.12032	123.419
0.12500	-0.30109E-01	0.87990E-01	108.890	0.17298E-01	0.11126E-01	0.32665E+00	0.12699	91.530
0.13281	0.58703E-01	0.51972E-01	41.519	0.12294E-01	0.10982E-01	0.32242E+00	0.12951	63.884
0.14063	0.40904E-01	0.18010E-02	2.521	0.33528E-02	0.91653E-02	0.26909E+00	0.14050	-7.290
0.14844	0.30947E-01	-0.70475E-01	-66.293	0.11849E-01	0.15156E-01	0.44497E+00	0.15306	-109.971
0.15625	-0.11379E+00	-0.46735E-01	-157.672	0.30265E-01	0.21262E-01	0.62426E+00	0.15745	-162.265
0.16406	-0.81621E-01	0.64612E-01	141.634	0.21673E-01	0.26136E-01	0.76736E+00	0.16368	154.253
0.17188	-0.53185E-01	0.10201E+00	117.536	0.26470E-01	0.22484E-01	0.66013E+00	0.17160	107.110
0.17969	0.59425E-01	0.78249E-01	52.786	0.19308E-01	0.16584E-01	0.48689E+00	0.17615	99.469
0.18750	-0.39684E-01	0.20275E-01	152.938	0.39718E-02	0.26048E-01	0.76476E+00	0.19105	70.274
0.19531	0.69494E-01	0.15034E+00	65.191	0.54863E-01	0.34169E-01	0.10032E+01	0.19834	30.276
0.20313	0.12353E+00	-0.81098E-01	-33.286	0.43671E-01	0.40105E-01	0.11775E+01	0.20098	30.538

0.21094	-0.10310E+00	-0.16191E-01	-171.074	0.21782E-01	0.35767E-01	0.10501E+01	0.21080	31.285
0.21875	0.45453E-01	0.13732E+00	71.686	0.41847E-01	0.32961E-01	0.96773E+00	0.21981	50.695
0.22656	0.12803E+00	-0.35156E-01	-15.355	0.35253E-01	0.28363E-01	0.83273E+00	0.22345	14.627
0.23438	-0.16131E-01	-0.61103E-01	-104.789	0.79877E-02	0.15335E-01	0.45025E+00	0.22886	-26.431
0.24219	0.93478E-02	0.35991E-01	75.441	0.27655E-02	0.85621E-02	0.25138E+00	0.24430	-45.588
0.25000	0.74415E-01	-0.43921E-01	-30.550	0.14933E-01	0.10463E-01	0.30720E+00	0.25272	-63.274
0.25781	-0.44719E-01	-0.69610E-01	-122.718	0.13691E-01	0.95861E-02	0.28145E+00	0.25379	-75.196
0.26563	-0.18044E-02	0.79995E-02	102.711	0.13450E-03	0.85054E-02	0.24972E+00	0.26501	-130.834
0.27344	-0.53728E-01	-0.54395E-01	-134.647	0.11691E-01	0.75556E-02	0.22183E+00	0.27713	161.984
0.28125	-0.19829E-01	0.70905E-01	105.624	0.10841E-01	0.10135E-01	0.29757E+00	0.28027	-143.588
0.28906	0.12616E-01	-0.61459E-01	-78.400	0.78728E-02	0.11190E-01	0.32854E+00	0.29000	157.949
0.29688	-0.81954E-01	0.26672E-01	161.972	0.14856E-01	0.91613E-02	0.26897E+00	0.29599	178.476
0.30469	-0.31213E-01	0.37462E-01	129.801	0.47553E-02	0.12560E-01	0.36876E+00	0.30535	122.762
0.31250	0.11153E-01	0.94392E-01	83.262	0.18069E-01	0.92538E-02	0.27169E+00	0.31255	77.274
0.32031	0.45158E-01	-0.20725E-01	-24.653	0.49375E-02	0.97914E-02	0.28748E+00	0.31720	74.239
0.32813	-0.20702E-01	0.52493E-01	111.523	0.63683E-02	0.64990E-02	0.19081E+00	0.32943	30.516
0.33594	0.61198E-01	0.18717E-01	17.006	0.81911E-02	0.64363E-02	0.18897E+00	0.33528	43.371
0.34375	0.47298E-01	0.11730E-01	13.928	0.47495E-02	0.10571E-01	0.31035E+00	0.34636	-11.862
0.35156	0.69225E-01	-0.67775E-01	-44.394	0.18771E-01	0.13065E-01	0.38360E+00	0.35374	-58.012
0.35938	-0.29353E-01	-0.83524E-01	-109.363	0.15676E-01	0.14818E-01	0.43504E+00	0.35783	-95.083
0.36719	-0.57048E-01	-0.41815E-01	-143.759	0.10006E-01	0.17038E-01	0.50022E+00	0.36868	-140.975
0.37500	-0.10805E+00	-0.32264E-01	-163.374	0.25431E-01	0.42426E-01	0.12456E+01	0.38002	161.266
0.38281	-0.12650E+00	0.17297E+00	126.179	0.91841E-01	0.74130E-01	0.21765E+01	0.38561	97.133
0.39063	0.20380E+00	0.10500E+00	27.258	0.10512E+00	0.79528E-01	0.23349E+01	0.38898	60.454
0.39844	-0.15100E-02	-0.14426E+00	-90.600	0.41625E-01	0.50844E-01	0.14928E+01	0.39335	-4.985
0.40625	-0.47250E-01	0.25737E-01	151.423	0.57898E-02	0.17548E-01	0.51520E+00	0.40085	-134.422
0.41406	-0.46454E-01	0.21365E-01	155.302	0.52289E-02	0.66509E-02	0.19527E+00	0.41529	119.020
0.42188	0.36519E-01	0.55977E-01	56.879	0.89341E-02	0.50699E-02	0.14885E+00	0.41973	107.477
0.42969	-0.18607E-01	0.13308E-01	144.428	0.10466E-02	0.81163E-02	0.23829E+00	0.43143	42.031
0.43750	0.82141E-01	0.20902E-01	14.277	0.14368E-01	0.10191E-01	0.29919E+00	0.44111	-138.238
0.44531	-0.74903E-01	-0.44361E-01	-149.364	0.15157E-01	0.20625E-01	0.60555E+00	0.44758	54.420
0.45313	0.57232E-01	0.11358E+00	63.256	0.32350E-01	0.22374E-01	0.65689E+00	0.45364	-30.056
0.46094	0.47872E-01	-0.86690E-01	-61.092	0.19614E-01	0.19726E-01	0.57915E+00	0.45762	32.269
0.46875	-0.60037E-01	0.15704E-02	178.502	0.72139E-02	0.12083E-01	0.35477E+00	0.46655	-146.226
0.47656	-0.14940E-01	0.66992E-01	102.572	0.94224E-02	0.86125E-02	0.25286E+00	0.47716	97.481
0.48438	0.62521E-01	0.26302E-01	22.816	0.92012E-02	0.64708E-02	0.18998E+00	0.48090	62.773
0.49219	0.94178E-02	0.17485E-01	61.693	0.78888E-03	0.71055E-02	0.20862E+00	0.49297	16.567
0.50000	0.75254E-01	0.0	0.0	0.11326E-01	0.11326E-01	0.33254E+00	0.50000	0.0
THE SUMS ARE				0.10057E+01	0.10057E+01	0.29528E+02		

ALSO NOTE THAT THE FIRST AND LAST 1 POINT(S) OF THE SMOOTHED POWER, AMPL. SQ., PHASE
AND WEIGHTED FREQUENCY COLUMNS HAVE NOT BEEN SMOOTHED

Table B.18. Sample print-out of MEM output with the Akaike FPE chosen automatically.

```
JOB NUMBER     4 WITH    68 VALUES USED OF WHICH     0 VALUES ARE PLOTTED AND/OR PRINTED; THE SIGN FOR THE FFT IS -1.0


RAW DATA MEAN =     -11.407 RAW DATA VARIANCE =        14.681 RAW DATA STANDARD DEVIATION =      3.832

THE NUMBER OF FILTER COEFFICIENTS WILL BE CHOSEN AUTOMATICALLY. THE EXPANSION FACTOR IS    1. DT = 1.000


THE PROCESS OF CONVERGENCE TO MINIMUM FPE NOW FOLLOWS
     M        FPE(M)           P(M)

     1     0.10250E+01     0.96523E+00
     2     0.10209E+01     0.93224E+00
     3     0.10303E+01     0.90855E+00
     4     0.98500E+00     0.81868E+00
     5     0.93978E+00     0.76866E+00
     6     0.96796E+00     0.76840E+00
     7     0.99518E+00     0.76772E+00
     8     0.10164E+01     0.76142E+00
     9     0.10463E+01     0.76113E+00
    10     0.10769E+01     0.75550E+00
    11     0.11101E+01     0.75357E+00
    12     0.11183E+01     0.72601E+00
    13     0.11083E+01     0.70071E+00
    14     0.11414E+01     0.69613E+00
    15     0.11614E+01     0.68918E+00
    16     0.11986E+01     0.68913E+00
    17     0.12384E+01     0.68541E+00
    18     0.12742E+01     0.68123E+00
    19     0.13050E+01     0.67802E+00
    20     0.13475E+01     0.67802E+00
    21     0.13720E+01     0.66030E+00
    22     0.14181E+01     0.65260E+00
    23     0.14667E+01     0.65045E+00
    24     0.15142E+01     0.64864E+00
    25     0.15611E+01     0.64319E+00
    26     0.16113E+01     0.63319E+00
    27     0.16758E+01     0.62638E+00
    28     0.17273E+01     0.61907E+00
    29     0.17908E+01     0.61906E+00
    30     0.18658E+01     0.59151E+00
    31     0.19115E+01     0.58390E+00
```

FOR M = 5 WE HAVE FPE(M) = 0.93978E+00 AND P(M) = 0.76866E+00

THE FILTER COEFFICIENTS ARE
-0.22257 -0.23042 -0.16659 -0.24027 0.24719

FPE(0) = 0.10298E+01 FPE(M) = 0.93978E+00 RELATIVE VALUE (RFPE(M)) = 0.91261E+00

FREQUENCY	POWER
0.0	0.29556E+00
0.00735	0.29549E+00
0.01471	0.29530E+00
0.02206	0.29515E+00
0.02941	0.29524E+00
0.03676	0.29584E+00
0.04412	0.29728E+00
0.05147	0.29993E+00
0.05882	0.30422E+00
0.06618	0.31061E+00
0.07353	0.31965E+00
0.08088	0.33199E+00
0.08824	0.34845E+00
0.09559	0.37006E+00
0.10294	0.39822E+00
0.11029	0.43479E+00
0.11765	0.48234E+00
0.12500	0.54445E+00
0.13235	0.62615E+00
0.13971	0.73448E+00
0.14706	0.87901E+00
0.15441	0.10719E+01
0.16176	0.13258E+01
0.16912	0.16450E+01
0.17647	0.20048E+01
0.18382	0.23230E+01
0.19118	0.24704E+01
0.19853	0.23709E+01
0.20588	0.20851E+01
0.21324	0.17405E+01
0.22059	0.14259E+01
0.22794	0.11723E+01
0.23529	0.97877E+00
0.24265	0.83438E+00
0.25000	0.72766E+00
0.25735	0.64940E+00
0.26471	0.59287E+00
0.27206	0.55338E+00

0.27941	0.52781E+00
0.28676	0.51425E+00
0.29412	0.51172E+00
0.30147	0.52012E+00
0.30882	0.54017E+00
0.31618	0.57359E+00
0.32353	0.62329E+00
0.33088	0.69391E+00
0.33824	0.79253E+00
0.34559	0.92977E+00
0.35294	0.11211E+01
0.36029	0.13877E+01
0.36765	0.17530E+01
0.37500	0.22240E+01
0.38235	0.27414E+01
0.38971	0.31170E+01
0.39706	0.31181E+01
0.40441	0.27381E+01
0.41176	0.22079E+01
0.41912	0.17235E+01
0.42647	0.13486E+01
0.43382	0.10758E+01
0.44118	0.88066E+00
0.44853	0.74081E+00
0.45588	0.64001E+00
0.46324	0.56715F+00
0.47059	0.51480E+00
0.47794	0.47807E+00
0.48529	0.45376E+00
0.49265	0.43989E+00
0.50000	0.43539E+00

Table B.19. Sample print-out of MEM output, bypassing the Akaike FPE.

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JOB NUMBER      5 WITH    68 VALUES USED OF WHICH     0 VALUES ARE PLOTTED AND/OR PRINTED; THE SIGN FOR THE FFT IS -1.0


RAW DATA MEAN =     -11.407 RAW DATA VARIANCE =        14.681 RAW DATA STANDARD DEVIATION =      3.832

THE NUMBER OF FILTER COEFFICIENTS CALCULATED IS   25. THE EXPANSION FACTOR IS    1. DT = 1.000


FOR M =     25 WE HAVE FPE(M) =      0.0          AND P(M) =      0.64319E+00

THE FILTER COEFFICIENTS ARE
  -0.17926  -0.26597  -0.18894  -0.16236   0.21396   0.09397
   0.04694   0.05879   0.01867   0.03632   0.05744  -0.17488
   0.21697  -0.00964   0.13007   0.07014   0.06285   0.03033
  -0.13243  -0.08951  -0.18680  -0.11362  -0.04209  -0.03586
   0.09166

FPE(0) =      0.10298E+01 FPE(M) =      0.0          RELATIVE VALUE (RFPE(M)) =     0.0

            FREQUENCY           POWER

             0.0              0.30455E+00
             0.00735          0.34695E+00
             0.01471          0.46025E+00
             0.02206          0.45252E+00
             0.02941          0.27420E+00
             0.03676          0.16674E+00
             0.04412          0.12838E+00
             0.05147          0.13093E+00
             0.05882          0.18363E+00
             0.06618          0.37064E+00
             0.07353          0.69687E+00
             0.08088          0.37902E+00
             0.08824          0.18584E+00
             0.09559          0.13283E+00
             0.10294          0.13436E+00
             0.11029          0.19126E+00
             0.11765          0.40383E+00
             0.12500          0.11993E+01
             0.13235          0.13550E+01
             0.13971          0.72892E+00
             0.14706          0.59711E+00
             0.15441          0.76410E+00
```

0.16176	0.13875E+01
0.16912	0.21916E+01
0.17647	0.17445E+01
0.18382	0.13929E+01
0.19118	0.16013E+01
0.19853	0.25151E+01
0.20588	0.32353E+01
0.21324	0.22460E+01
0.22059	0.15229E+01
0.22794	0.12137E+01
0.23529	0.95452E+00
0.24265	0.66857E+00
0.25000	0.47436E+00
0.25735	0.39089E+00
0.26471	0.39074E+00
0.27206	0.46204E+00
0.27941	0.58592E+00
0.28676	0.68905E+00
0.29412	0.72847E+00
0.30147	0.76607E+00
0.30882	0.80576E+00
0.31618	0.72920E+00
0.32353	0.55495E+00
0.33088	0.43339E+00
0.33824	0.40380E+00
0.34559	0.47015E+00
0.35294	0.66309E+00
0.36029	0.97045E+00
0.36765	0.12115E+01
0.37500	0.15097E+01
0.38235	0.29402E+01
0.38971	0.12578E+02
0.39706	0.25403E+01
0.40441	0.66606E+00
0.41176	0.34211E+00
0.41912	0.26546E+00
0.42647	0.28709E+00
0.43382	0.42747E+00
0.44118	0.87080E+00
0.44853	0.18000E+01
0.45588	0.16295E+01
0.46324	0.10023E+01
0.47059	0.66870E+00
0.47794	0.45990E+00
0.48529	0.32376E+00
0.49265	0.25191E+00
0.50000	0.23025E+00